

How Market Ecology Explains Market Malfunction

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Standard approaches to the theory of financial markets are based on equilibrium and efficiency. Here we develop an alternative based on concepts and methods developed by biologists, in which the wealth invested in a financial strategy is like the population of a species. We study a toy model of a market consisting of value investors, trend followers and noise traders. We show that the average returns of strategies are strongly density dependent, i.e. they depend on the wealth invested in each strategy at any given time. In the absence of noise the market would slowly evolve toward an efficient equilibrium, but the large statistical uncertainty in profitability makes this noisy and uncertain. Even in the long term, the market spends extended periods of time far from perfect efficiency. We show how core concepts from ecology, such as the community matrix and food webs, apply to markets. The wealth dynamics of the market ecology explains how market inefficiencies spontaneously occur and gives insight into the origins of excess price volatility and deviations of prices from fundamental values.

market ecology | market efficiency | agent-based modeling

Why do markets malfunction? According to the theory of market efficiency, markets always function perfectly: Prices always reflect fundamental values and they only change when there is new information that affects fundamental values. Thus, by definition, any problems with price setting are caused by factors outside the market. Empirical evidence suggests otherwise. Large price movements occur even when there is very little new information (1) and prices often deviate substantially from fundamental values (2). This indicates that to understand how and why markets malfunction we need to go beyond the theory of market efficiency.

Here we build on earlier work (3–7) and develop the theory of market ecology, which provides just such an alternative. This approach borrows concepts and methods from biology and applies them to financial markets. Financial trading strategies are analogous to biological species. Plants and animals are specialists that evolve to fill niches that provide food; similarly, financial trading strategies are specialized decision making rules that evolve to exploit market inefficiencies. Trading strategies can be classified into distinct categories, such as technical trading, value investing, market making, statistical arbitrage and many others. The capital invested in a strategy is like the population of a species. Trading strategies interact with one another via price setting and the market evolves as the wealth invested in each strategy changes through time, and as old strategies fail and new strategies appear.

The theory of market ecology builds on the inherent contradictions in the theory of market efficiency. A standard argument used to justify market efficiency is that competition for profits by arbitrageurs should cause markets to rapidly evolve to an equilibrium where it is not possible to make excess profits based on publicly available information. But if there are no profits to be made, there are no incentives for arbitrageurs, so there is no mechanism to make markets

efficient. This paradox suggests that, while markets may be efficient in some approximate sense, they cannot be perfectly efficient (8). In contrast, under the theory of market ecology, trading strategies exploit market inefficiencies but, as new strategies appear and as the wealth invested in each strategy changes, the inefficiencies change as well. To understand how the market functions, it is necessary to understand how each strategy affects the market and how the interactions between strategies cause market inefficiencies to change with time. The theory of market ecology naturally addresses a different set of problems than the theory of market efficiency, and can be viewed as a complement rather than a substitute for it.

Here we study a stylized toy market model with three trading strategies. We approach the problem in the same way that an ecologist would study three interacting species. We study how the average returns of the strategies depend on the wealth invested in each strategy and how their wealth evolves through time under reinvestment, and how this endogenous time evolution causes the market to malfunction.

Unlike previous studies that use a market impact rule for price setting (3, 9), here we use market clearing. This provides a better model and in some cases leads to substantially different results. In contrast to market impact, under market clearing, all the properties of the market ecology depend strongly on the wealth invested in each strategy.

We show that evolution toward market efficiency is very slow. The expected deviations from efficiency are typically substantial, even in the long term, and cause extended deviations from fundamental values and excess volatility (which in extreme cases becomes market instability). Our study provides a simple example of how analyzing markets in these terms and tracking market ecologies through time could give

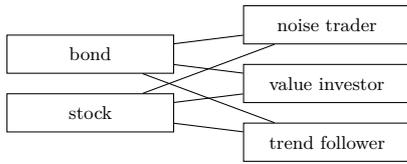
Significance Statement

We develop the mathematical analogy between financial trading strategies and biological species and show how to apply standard concepts from ecology to financial markets. We analyze the interactions of three stereotypical trading strategies in ecological terms, showing that they can be competitive, predator-prey or mutualistic, depending on the wealth invested in each strategy. The deterministic dynamics suggest that the system should evolve toward an efficient state where all three strategies make the same average returns. However, this happens so slowly and the evolution is so noisy that there are large fluctuations away from the efficient state, causing bursts of volatility and extended periods where prices deviate from fundamental values. This provides a conceptual framework that gives insight into the reasons why markets malfunction.

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Fig. 1. The three trading strategies correspond to noise traders, value investors and trend followers. They invest their capital in a stock and a bond. The mixture for each strategy changes with time as strategies accumulate wealth based on their historical performance.



regulators better insight into market behavior.

A. Model Description. The structure of the model is schematically summarized in Figure 1. There are two assets, a stock and a bond. The bond trades at a fixed price and yields $r = 1\%$ annually in the form of coupon payments that are paid out continuously. The stock pays a continuous dividend $D(t)$ that is modeled as an autocorrelated geometric Brownian motion, of the form

$$\begin{aligned} dD(t) &= gD(t)dt + \sigma D(t)dU(t), \\ dU(t) &= (1 - \omega)dZ(t) + \omega dU(t - \Delta t), \end{aligned} \quad [1]$$

where g is the average rate of dividend payments per unit of time t , σ is the variance, ω is the autocorrelation of the process, and Z and U are standard Wiener processes. We approximate the continuous processes by discrete processes with a time step equal to one day. We use estimates from market data by Lebaron (10), taking $g = 2\%$ per year for the growth rate of the dividend with a volatility of $\sigma = 6\%$. (See reference (11), for example, for a review of the empirical evidence on dividends).

We use market clearing to set prices. Each asset a has a fixed supply Q_a , but the excess demand $E(t)$ for the stock by each trading strategy varies in time. We allow the trading strategies to take short positions and to use leverage (i.e. to borrow in order to take a position in the stock that is larger than their wealth). We impose a strategy-specific leverage limit λ^* . Because we use leverage and because the strategies can have demand functions with unusual properties, market clearing is not always straightforward – see Materials and Methods.

The size of a trading strategy is given by its *wealth* $W(t)$, i.e. the capital invested in it at any given time. In ecology this corresponds to the population of a species, which is also called its abundance. Unless otherwise stated, we assume profits and losses are reinvested, so that the wealth of each strategy varies according to its cumulative performance.

A trading strategy is defined by its trading signal $\phi(t)$, which can depend on the price $p(t)$ and other variables, such as dividends and past prices. We modify ϕ by a tanh function to ensure that the excess demand is bounded and differentiable. A strategy's excess demand for the stock is

$$E(t) = \frac{W(t)\lambda^*}{p(t)} \left(\tanh(c \cdot \phi(t)) + \frac{1}{2} \right) - S(t-1), \quad [2]$$

where $S(t-1)$ is the number of shares of the stock held at the previous time step. The parameter $c > 0$ determines the aggressiveness of the response to the signal ϕ , and is strategy specific. When the signal of the strategy is zero, the agent is indifferent between the stock and the bond and splits its portfolio equally between the two (hence the factor of $1/2$).

The leverage $\lambda(t)$ of a strategy at any given time is $\frac{p(t)S(t)}{K(t)} = \lambda^* |\tanh(\phi(t)) + \frac{1}{2}|$. This equality holds when the market clears.

B. Investment Strategies. We study three typical trading strategies, which we call value investors, trend followers and noise traders. We make a representative agent hypothesis, treating each strategy as though it were only used by a single fund; however, these should be thought of as representing all investors using these strategies. We now describe each of them in turn.

Value Investors observe the dividend process, use a model to derive the value of the stock, and seek to hold more of the stock when it is undervalued and hold more of the bond when it is overvalued. The parameters of their model are estimated based on the historical dividends. However their model is inaccurate in that it contains estimation errors and it does not take the autocorrelation of the dividend process into account (i.e. they assume $\omega = 0$).

Following Gordon and Shapiro (12), the fundamental value $V(t)$ of a stock is

$$\begin{aligned} V(t) &= \int_{t+1}^{\infty} D(t+1)e^{\hat{g}\tau} e^{-k\tau} d\tau \\ &= \frac{D(t+1)}{k - \hat{g}}, k > \hat{g}. \end{aligned} \quad [3]$$

The parameter k is the discount rate, also called the *required rate of return*. It is the sum of the risk-free rate and a risk-premium investors expect for the additional risks associated with the stock. We follow (13) and use a fixed discount rate $k = 2\%$ based on the average rate of return implied by historical data.

We define the trading signal for the value investor as the difference in log prices between the estimated fundamental value $V(t)$ and the market price.

$$\phi_{VI}(t) = \log_2 V(t) - \log_2 p(t) \quad [4]$$

This strategy will enter into a long position when the proposed price is larger than the value estimated by the investor, and it will enter into a short position when the proposed price is higher than the estimated fundamental value. The use of the base two logarithm means that the value investor employs all of its assets when the stock is trading at half the perceived value (14).

Trend Followers expect that historical trends in returns continue into the short term future. Several variants exist in the literature, including the archetypal trend follower that we use here (15–18). There is evidence to suggest that trend-based investment strategies are profitable over long time horizons, and reference (19) argues that investors earn a premium for the liquidity risk associated with stocks with high momentum (momentum trading is a synonym for trend following).

The trend strategy we use extrapolates the trend in price between θ_1 and θ_2 time steps in the past as follows

$$\phi_{TF}(t) = \log_2 p(t-\theta_1) - \log_2 p(t-\theta_2), \quad \theta_1 < \theta_2. \quad [5]$$

We choose $\theta_1 = 1$ and $\theta_2 = 2$ and keep them fixed. This choice of parameters allows the trend follower to exploit the

autocorrelation that the dividends impart to prices. The trend followers' demand is a decreasing function of price. Trend followers will make profits if there is positive autocorrelation in the stock's returns, e.g. due to the dividend process.

Noise Traders represent non-professional investors who do not track the market closely. Their transactions are mostly for liquidity, but they are also somewhat aware of value, so that they are slightly more likely to buy when the market is undervalued and slightly more likely to sell when the market is overvalued. The signal function of our noise traders contains the product of the value estimate $V(t)$ (which we assume is the same as for the value investors) and a stochastic component $X(t)$,

$$\phi_{NT}(t) = \log_2 X(t)V(t) - \log_2 p(t). \quad [6]$$

The noise process $X(t)$ is an Ornstein-Uhlenbeck process, which has the form

$$dX(t) = \rho(\mu - X(t))dt + \gamma dW(t) \quad [7]$$

This process reverts to the long term mean $\mu = 1$ with reversion rate $\rho = 1 - \frac{1}{\sqrt{0.5}^{6 \times 252}}$, meaning the noise has a half life of 6 years, in accordance with the values estimated by Bouchaud (20). $W(t)$ is a Wiener process and $\gamma = 12\%$ is a volatility parameter, which is twice the volatility of the dividend process.

The parameters of the model are summarized in Table 5. We have chosen them for an appropriate compromise between realism and conceptual interest, e.g. so that each strategy has a region in the wealth landscape where it is profitable.

Results

C. Density Dependence. An ecology is density dependent if the characteristics of the ecology depend on the populations of the species, as is typically the case. Similarly, a market ecology is density dependent if its characteristics depend on the wealth invested in each strategy. The toy market ecology that we study here is strongly density dependent.

When the core ideas in this paper were originally introduced in reference (3), prices were formed using a market impact function, which translates the aggregate trade imbalance at any time into a shift in prices. This can be viewed as a local linearization of market clearing. The use of a market impact function suppresses density dependence and neglects nonlinearities that are important for understanding market ecologies.

In contrast, using market clearing we see strong density dependence. This is evident in Figure 2, which shows which strategy makes the highest profits as a function of the relative size of each of the three strategies. To control the size of each strategy we turn off reinvestment, and instead replenish the wealth of each strategy at each step as needed to hold it constant. We then systematically vary the wealth vector $\mathbf{W} = (W_{NT}, W_{VI}, W_{TF})$. We somewhat arbitrarily let $W_{NT} + W_{VI} + W_{TF} = 3 \times 10^8$, but we plot the relative wealth (as if the wealths sum to one). The results shown are averages over many long runs; to avoid transients we exclude the first 252 time steps, corresponding to one trading year.

Roughly speaking, the profitability of the dominant strategy divides the wealth landscape into four distinct regions. Trend followers dominate at the bottom of the diagram, where

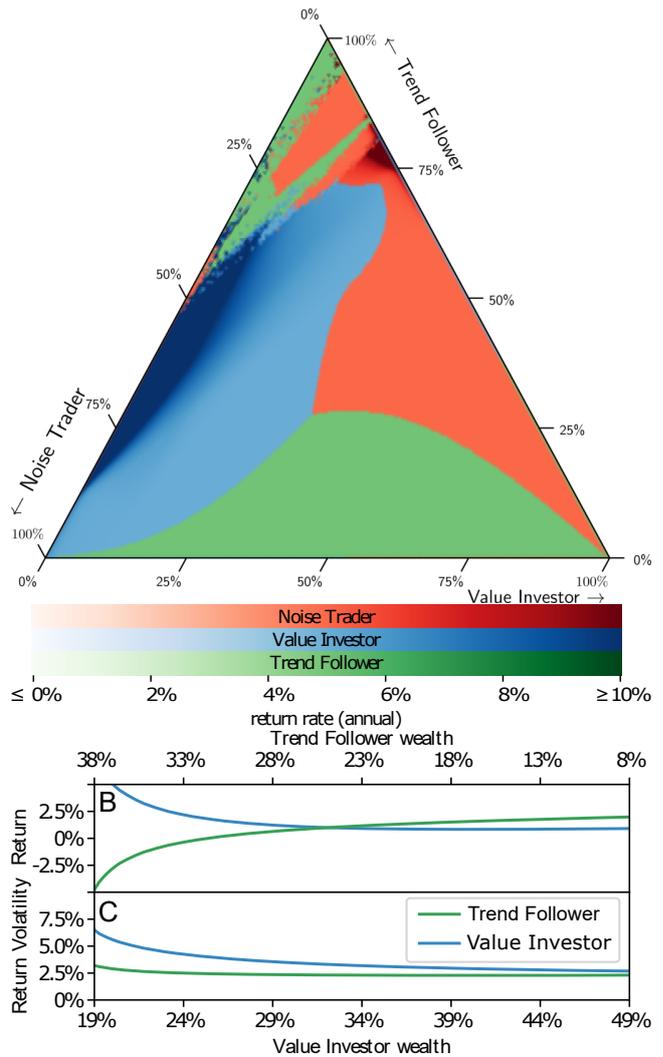


Fig. 2. The profitability of the dominant strategy in the wealth landscape. Panel A is a ternary plot which displays the returns achieved by the strategy with the largest return. The axes correspond to the relative wealth invested in each strategy. The top corner is pure trend followers, the left corner pure noise traders, and the right corner pure value investors. The color indicates the strategy with the highest returns at a given wealth vector \mathbf{W} . The regions colored in red correspond to the noise traders, blue regions to value investors, and green to the trend followers. The intensity of the color indicates the size of the average return. The upper box of panel B shows the average returns to value investors (blue) and trend followers (green) while holding the noise trader wealth at its equilibrium value of 42%. The lower box shows the volatility in the returns of each strategy. The horizontal axis is the relative wealth of the trend follower (top) and value investor (bottom).

their wealth is small. Value investors dominate on the left side of the diagram, where their wealth is small, and noise traders dominate on the right side of the diagram, where their wealth is small. There is an intersection point near the center where the returns of all three strategies are the same, corresponding to an efficient equilibrium. In addition, there is a complicated region at the top of the diagram, where no single strategy dominates. The turbulent behavior in this region comes about because the wealth invested by trend followers is large and the price dynamics are unstable.

A quantitative snapshot of the average returns and volatil-

ity is given in Figure 2B, where we hold the size of the noise traders constant at its equilibrium level of 42% and vary the wealth of the value investors and trend followers. The average return to both trend followers and value investors increases monotonically as their wealth decreases. The volatility of the returns of both strategies, in contrast, is a monotonic function of the wealth of the trend followers – higher trend follower wealth implies higher volatility. Although this is not shown here, the average return of the value investors increases strongly with the wealth of the noise traders; in contrast, the average return of the trend followers is insensitive to it.

D. Adaptation. We now investigate the dynamics of the ecology. To understand how the wealth of the strategies evolves through time, we allow reinvestment and plot trajectories corresponding to the average return from each point \mathbf{W} . This is done by averaging over many different runs. The result is shown in Figure 3A. Most of the wealth trajectories in the diagram evolve toward the efficient equilibrium in the center, where there is a fixed point where the wealths of the strategies no longer change. At the equilibrium the returns to the three strategies are all equal to $\pi = 2.09\%$, which is slightly better than simply buying and holding the stock. However, there is a region at the top of the diagram where the dynamics are more complicated and a region in the lower left corner where the ecology evolves toward the boundary of the simplex.

These results give the impression of a smooth evolution toward a state of market efficiency, but this is misleading. In fact the dynamics are very noisy, and stray very far from the deterministic dynamics suggested by Figure 3A. Tracking a few individual trajectories, as we do in Figure 3B, demonstrates that the dynamics are dominated by noise due to the statistical uncertainty in the performance of the strategies. The typical trajectories bear little correspondence to the deterministic trajectories of Figure 3A. Furthermore, the evolution of the wealth is exceedingly slow: Each trajectory spans 200 years of simulated time. There are substantial changes in the relative wealth taking place over time scales that are longer than a century, and the convergence to the equilibrium point seen in panel A is at best weak and uncertain.

The long time scale for the approach to market efficiency should not be surprising. As originally pointed out in reference (3), the large statistical uncertainty in the performance of a trading strategy implies a long time scale to attain efficiency. A common way to measure the performance of a trading strategy is in terms of the ratio of the mean to the standard deviation of its returns, which is called the Sharpe ratio \mathcal{S} . In the ideal case of a stationary market and I.I.D. normally distributed returns, the time required to detect excess performance $\Delta\mathcal{S}$ with a statistical significance of s standard deviations is approximately $\tau = (s/\Delta\mathcal{S})^2$. To take an example, a buy and hold of the S&P index has a Sharpe ratio of roughly $\mathcal{S} = 0.5$. It thus requires roughly 400 years to confirm the performance of a strategy that outperforms the index by 20% at two standard deviations of statistical significance. Furthermore, as shown in reference (21), because the rate of approach to market efficiency slows down as it is approached, it follows a power law of the form $t^{-\alpha}$, where $0 \leq \alpha \leq 1$. For large times this is much slower than an exponential.

To better understand the long-term evolution, we sample

the space of initial wealth uniformly, simulate the ecological dynamics under reinvestment, and record the final wealth after 200 years, as shown in Figure 3C. While the fixed point equilibrium is contained in the region where the ecology is most likely to be found, it is not in the center of this region, and the deviations in the relative wealth of the strategies from the equilibrium are substantial, often more than 20%.

The autocorrelation of price returns is an indicator of market efficiency. Efficient price returns should have an autocorrelation that is reasonably close to zero (close enough that it is not possible to make statistically significant excess profits). In Figure 3D we plot the autocorrelation of returns across the wealth landscape. There is a striking white band across the center of the simplex, corresponding to zero autocorrelation. This happens when trend followers invest about 40% of the total wealth, thereby eliminating the autocorrelation coming from the dividend process.

E. Community matrix. The community matrix is a tool used in ecology to describe the pairwise effects of the population of species j on the population growth rate of species i (22, 23). As originally pointed out by Farmer (3), who called it the gain matrix, analogous quantity is also useful for interpreting the behavior of market ecologies. Assuming differentiability, let $\Delta W_i(t) = dW_i/dt$ be the profits per unit time, so that $\pi_i(t) = \Delta W_i(t)/W(t)$ is the return to strategy i , and let the relative wealth $w_i(t) = W_i/W_T$, where W_T is the total wealth. The analogue of the community matrix for market ecologies is

$$G_{ij} = \frac{\partial \Delta W_i}{\partial W_j} = \frac{\partial \pi_i}{\partial w_j}. \quad [8]$$

This has units of one over time. The wealth $W_i(t)$ invested in strategy i replaces the population size of a species. The second equation makes explicit the sense in which the terms in the community matrix are like elasticities in economics, i.e. they measure the response of the returns to relative changes in wealth. The possible pairwise interactions between strategies can be classified according to the sign of G_{ij} . If both G_{ij} and G_{ji} are negative, then strategies i and j are competitive; if G_{ij} is positive and G_{ji} is negative, then there is a predator-prey interaction, with i the predator and j the prey; and if both G_{ij} and G_{ji} are positive, then there is a mutualistic interaction (24).

Because we do not have a differentiable model for our toy market ecology, we compute the community matrix numerically using finite differences (see Materials and Methods). The community matrix is strongly density dependent. If we compute the community matrix near the equilibrium point in the center of the simplex, we get the result shown in Table 1.

The diagonal entries are all negative, indicating that the strategies are competitive with themselves. This means that their average returns diminish as the strategy gets larger, causing what is called *crowding* in financial markets. We already observed this in Figure 2. Interestingly, however, the size of the diagonal terms varies considerably, from -0.89 for noise traders to -19.3 for trend followers. This means that we should expect trend followers to experience crowding more strongly than noise traders.

All the other entries are positive, indicating mutualism. This implies that every strategy benefits from an increase in the wealth of any of the other strategies. While we initially found it surprising that all the strategies could have

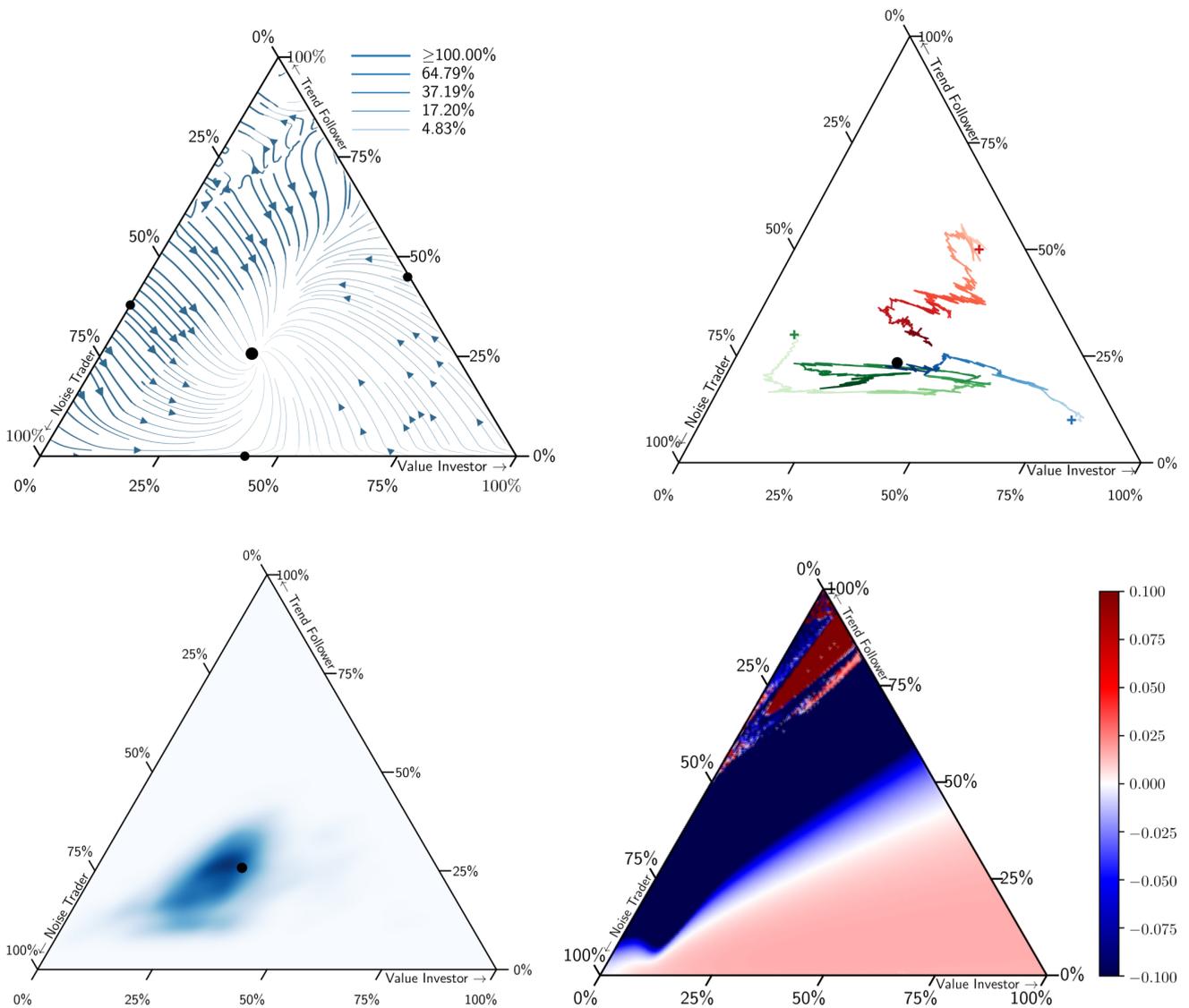


Fig. 3. Profit dynamics as a function of wealth. **A** shows how wealth evolves *on average* through time under reinvestment. The intensity of the color denotes the rate of change. **B** shows sample trajectories for a few different initial values of the wealth vector, making it clear that the trajectories are extremely noisy due to statistical uncertainty, so that the deterministic dynamics of panel A is a poor approximation. The visualization displays three different initial wealth vectors, each color-coded. The marker + indicates the initial wealth. The trajectories with the same color follows the system for $T = 200$ years and color saturation increases with time. Starting from uniformly distributed initial conditions, **C** displays a density map of the asymptotic wealth distribution after 200 years. The system is initialized at random with a uniformly distributed wealth vector and then allowed to freely evolve for 200 years. The darkness is proportional to density. The black dot is the equilibrium point from Panel A. Panel **D** displays the autocorrelation in the realized prices.

Table 1. Estimated community matrix near the equilibrium at $W = (NT = 0.43, VI = 0.34, TF = 0.23)$.

G_{ij}	NT	VI	TF
NT	-0.89%	0.89%	0.82%
VI	26.6%	-10.6%	22.4%
TF	11.1%	15.2%	-19.3%

Table 2. Estimated community matrix near $W = (NT = 0.26, VI = 0.55, TF = 0.19)$.

G_{ij}	NT	VI	TF
NT	-0.46%	0.40%	0.36%
VI	8.94%	-0.77%	-1.89%
TF	6.81%	6.87%	-9.65%

mutualistic interactions with each other, on reflection this makes sense: the ecology is by definition efficient at the equilibrium, and driving any of the strategies away from equilibrium creates an inefficiency that provides a profit opportunity for the other two strategies. (It is not clear to us whether this is specific to this particular set of strategies, or whether a richer set of strategies would display more complicated behavior near an efficient equilibrium).

The community matrix is density dependent. If we compute the community matrix at the wealth vector given in Table 2, where the value investors are dominant, there is a shift in the pairwise community relations. As before, all of the terms in the row corresponding to the noise traders are small, indicating that the noise traders are not strongly affected by other strategies, and that they compete only weakly with themselves. This should not be surprising – the noise traders’ strategy is mostly random, and is less influenced by prices than the other two strategies. Value investors, who have the majority of the wealth in this case, still strongly benefit from an increase in the wealth of noise traders (though less so than at the equilibrium). However, there is now a negative term in the second row, corresponding to the effect of trend followers. In contrast, from the third row we see that trend followers benefit from an increase in the wealth of both noise traders and value investors, implying that trend followers now prey on value investors. Other variations in community relationships can be found at different points in the wealth landscape, illustrating density dependence.

The Lotka-Volterra equations, which describe how the populations in an idealized predator-prey system evolve through time, are perhaps the most famous equations in population biology. Their surprising result is that at some parameter values they have solutions that oscillate indefinitely. Using the assumption of no density dependence, Farmer derived Lotka-Volterra equations for market ecologies (18). Our results here indicate that the density dependence in this system is so strong that simple Lotka-Volterra equations are a poor approximation, at least for this system. The existence of oscillating solutions in financial markets remains an open question.

F. Food Webs and Trophic Level. The food web provides an important conceptual framework for understanding the interactions between species. If lions eat zebras and zebras eat grass, then the population of lions is strongly affected

by the density of grass, and similarly the density of grass depends on the population of lions, even though lions have no direct interactions with grass. The trophic level of a species is by definition one level higher than what it eats, so in this idealized system grass has trophic level one, zebras have trophic level two and lions trophic level three.

The existence of animals with more complicated diets, such as omnivores and detritivores, means that real food webs are never this simple. If we let A_{ij} be the share of species j in the diet of species i , then the trophic level T_i of species i can be computed by the relation

$$T_i = 1 + \sum_j A_{ij} T_j. \quad [9]$$

The resulting trophic levels are typically not integers, but they still provide a useful way to think about the role that a given species plays in the ecology.

We can also compute trophic levels for the strategies in a market ecology. We define the analogous quantity A_{ij} as the fraction of the returns of strategy i that can be attributed to the presence of strategy j . We do this by simply comparing the returns of strategy i at wealth W to those when strategy j is removed, i.e. when $W_j = 0$ but all the other wealths remain the same. In mathematical terms,

$$A_{ij} = \max[0, \pi_i(W_1, \dots, W_j, \dots, W_N) - \pi_i(W_1, \dots, 0, \dots, W_N)]. \quad [10]$$

The maximum is taken so that A_{ij} is never negative. For computing the trophic levels we only care about the strategies that i benefits from, not those that cause it losses.

Equations (9) and (10) allow us to compute trophic levels for each of the strategies. At the equilibrium point, for example, the trophic levels are (1, 2, 3). In order to better understand the density dependence, we compute trophic levels at each point in the wealth landscape. For three strategies there are $3! = 6$ possible orderings of the trophic levels. We display the ordering of the trophic levels across the wealth landscape in Figure 4.

The computation of trophic levels is complicated by the fact that for some wealth vectors there are cycles in the food web. For example, for $W = (0.05, 0.15, 0.80)$, value investors gain from the noise traders, trend followers exploit the autocorrelation induced by the value investors, and the noise traders in turn benefit when the trend follower concentration increases (generating more volatility), to complete a cycle. When this happens the trophic levels become unrealistically large, equation (9) may not converge, and the trophic levels become undefined. Cycles are not unique to markets – they can also occur in biology, for example due to cannibalism or detritivores.

From Figure 4C we know that the most important part of the wealth landscape is the region around the efficient equilibrium. A comparison of Figure 5 to Figure 4C makes it clear that the system spends most of its time in the region in which the trophic levels are ordered as (Noise Trader, Value Investor, Trend Follower), as they are at the equilibrium point. While this is the dominant region, excursions into other regions are not uncommon.

Given that the noise trader strategy was constructed in order to represent a non-professional investor, it is natural that under normal circumstances it sits at the lowest trophic level. We have assumed here that the initial wealth endowments of

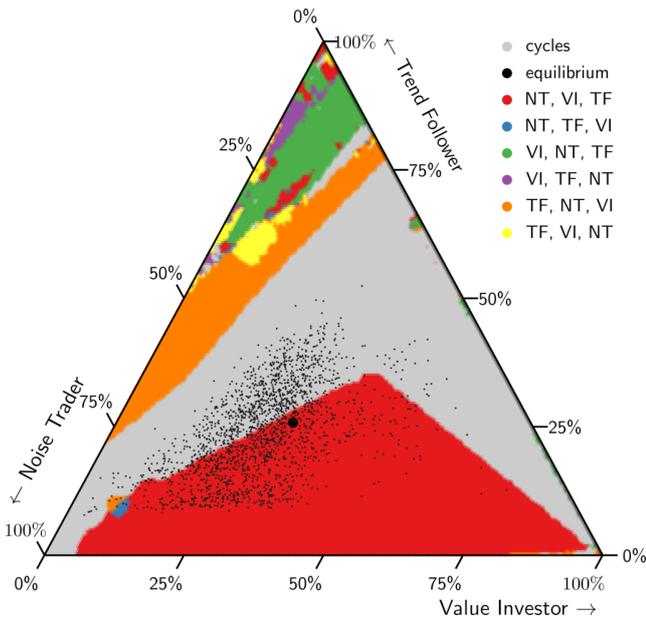


Fig. 4. A survey of the trophic levels across the wealth landscape. We color the diagram according to the ordering of the trophic levels of the three strategies (see legend). Red, for example, denotes the dominant zone where the noise traders have the lowest trophic level and trend followers have the highest trophic level (with value investors in the middle). In the grey region there are cycles where the trophic levels become undefined. The black dots correspond to samples of the wealth vector after 200 years, as shown in Figure 3(c). The system spends most of its time in the grey and red zones.

all the strategies are fixed, and the wealth changes only due to reinvestment. However in reality there will be other inflows of capital. The noise trader represents market participants who use the market for other purposes, such as liquidity or as a default place to put their excess capital. It is thus likely that the noise trader strategy will have an influx of external capital regardless of its profitability, and thus might naturally act as the “basal species” in the ecology. It is reassuring that the least sophisticated strategy emerges as the one that has the lowest trophic level under normal circumstances.

G. How ecological dynamics cause market malfunction. The wealth dynamics of the market ecology help explain why the market malfunctions and illuminate the origins of excess volatility and *mispricing*, i.e. deviations of prices from fundamental values. Volatility and mispricings are both functions of time – there are eras where they are large and eras where they are small. Volatility tends to vary intermittently, with periods of low volatility punctuated by bursts of high volatility – this behavior is called clustered volatility. The standard explanation for clustered volatility is fluctuating agent populations (15, 25, 26). Our analysis reinforces this explanation, but gives more insight into its causes. Clustered volatility can also be caused by leverage (27). While we observe that clustered volatility increases with increasing leverage, we have not investigated this in detail here.

Figure 5A presents the variation of the volatility across the wealth landscape. The landscape can roughly be divided into two regions. On the lower right there is a flat low volatility “plain” occupying most of the landscape. On the upper left there is a high volatility region, with a sharp boundary

between the two. As we will now show, excursions into the high volatility region cause clustered volatility. A similar story holds for mispricing.

Figure 5A shows a sample trajectory that begins at the efficient equilibrium and spans 200 years. The statistical fluctuations in the performance of the three strategies acts as noise, causing large excursions away from equilibrium. The trajectory mostly remains on the volatility plain, but there are several epochs where it ventures into the high volatility region causing bursts of high volatility.

The wealth dynamics have strong explanatory power for

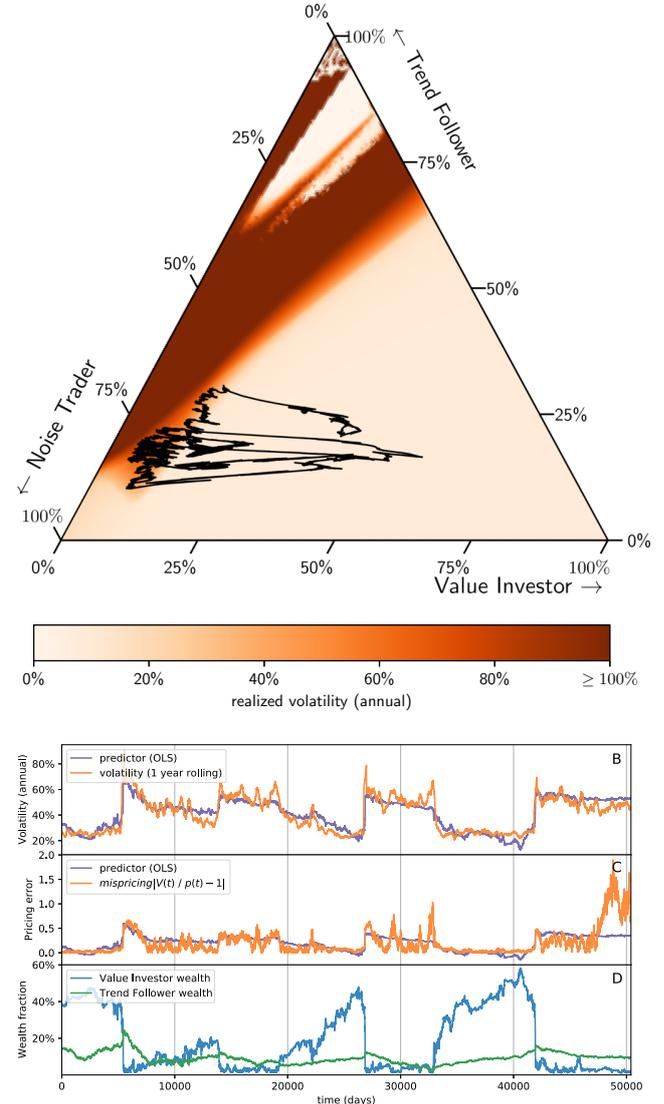


Fig. 5. How fluctuations in the ecology cause market malfunctions. Panel A gives a color map of the price volatility over the wealth landscape; the volatility is low and constant throughout the lower right part of the diagram, where the system spends most of its time, but there is a high volatility region running across the upper left. A sample trajectory spanning 200 years beginning at the efficient equilibrium is shown in black. The noise caused by statistical fluctuations in performance causes large deviations from equilibrium and excursions into the high volatility region. Panel B shows the volatility of this trajectory as a function of time, plotted against the predicted volatility from equation (11). Panel C shows the actual mispricing plotted against the predicted mispricing from equation (12). Panel D shows the wealth of the value investors and trend followers.

Table 3. Multivariate regressions with volatility and mispricing as dependent variables and the funds' wealth as independent variables.

volatility		
	$R^2 = 0.79$	observations: 50,397
independent variable	coefficient	t
noise trader	2.4	10
value investor	-68	-249
trend follower	107	169
mispricing		
	$R^2 = 0.33$	observations: 50,397
independent variable	coefficient	t
noise trader	-0.15	-18
value investor	-1.02	-107
trend follower	1.5	69

both mispricing and volatility. This is illustrated in Table 3, where we perform regressions of the strategies' wealth against volatility using daily values for the time series shown in Figure 5A. For volatility $R^2 = 0.79$ and for mispricing $R^2 = 0.33$. In both cases the value investor wealth and the trend follower wealth have large coefficients (in absolute value) and the fit is overwhelmingly statistically significant. The noise trader is also highly statistically significant but the coefficients and the t-statistics are more than an order of magnitude smaller. In Figure 5B and C we compare a time series of the predicted volatility and predicted mispricing,

$$\hat{v} = -68w_{vi} + 107w_{tf} + 2.4w_{nt}, \quad [11]$$

$$\hat{m} = -1.02w_{vi} + 1.5w_{tf} - 0.15w_{nt}, \quad [12]$$

against the actual values. The series match very well. Note that in both cases the coefficient for trend followers is positive, indicating that they drive instabilities, and the coefficient for value investors is negative, indicating their stabilizing influence.

Nonetheless, due to their effect on the population of value investors, the net effect of the trend followers on market malfunctions is not obvious. In Figure 5D we plot the wealth of value investors and trend followers. The strong mutualism predicted by the community matrix is clearly evident from the fact that the wealth of trend followers and value investors rises and falls together. However, their dynamics are quite different – there are several precipitous drops in the value investors' wealth, whereas the trend followers tend to take more gradual losses. As predicted, the highest volatility episodes happen when the value investors' wealth drops sharply while the trend followers' wealth is high.

Discussion

Our analysis here demonstrates how understanding fluctuations of the wealth of the strategies in the ecology can help us predict market malfunctions such as mispricings and endogenously generated clustered volatility. The toy model that we study here is simple and highly stylized, but it illustrates how one can import ideas from ecology to better understand financial markets. Our analysis of this model illustrates several properties of market ecologies that we hypothesize are likely to be true in more general settings.

This model gives important insights into how deviations from market efficiency occur and how they affect prices. While the market may be close to efficiency in the sense that the excess returns to any given strategy are small,

there can nonetheless be substantial deviations in the wealth of different strategies, that can cause excess volatility and market instability.

Market ecology is a complement rather than a substitute for the theory of market efficiency. There are circumstances, such as pricing options, where market efficiency is a useful hypothesis. Market ecology, in contrast, provides insight into how and why markets deviate from efficiency, and what the consequences of this are. It can be used to explain the time dependence in the returns of trading strategies, and in some cases it can be used to explain market malfunctions. One of our main innovations here is to demonstrate how to compute the community matrix and the trophic web, which provide insight into the interactions of strategies.

There are so far only a few examples of empirical studies of market ecologies (28, 29). This is because such a study requires counterparty identifiers on transactions in order to know who traded with whom. Trying to study a market ecology without such data is like trying to study a biological ecology in which one can observe that an animal ate another animal without any information about the types of animals involved. Unfortunately, for markets such data is difficult for most researchers to obtain.

Regulators potentially have access to the balance sheets of all market participants, which can allow them to track the ecology of the markets they regulate in detail. Ideas such as those presented here could provide valuable insight into when markets are in danger of failure, and make it possible to construct models for the ecological effect of innovations, e.g. the introduction of new types of assets such as mortgage-backed securities.

One of our most striking results is that the approach to efficiency is highly uncertain and exceedingly slow. As already pointed out, this should be obvious from a straightforward statistical analysis, but it is not widely appreciated. Our results demonstrate this dramatically and they indicate that, even in the long-term, we should expect large deviations from efficiency.

There are many possible extensions to this work. An obvious follow up is to explore a larger space of strategies, or to let new strategies evolve in an open-ended way through time. Does the process of strategy innovation tend to stabilize or destabilize markets? Another follow up is to construct a model that is empirically validated against data with counterparty identifiers. Our analysis here provides concepts and methods that could be used to interpret the behavior of real world examples.

Materials and Methods

Accounting and Balance Sheets. The funds in our model use a stylized balance sheet that is presented in table 4. External investors endow the fund with a certain amount of equity capital E , in the forms of cash C in dollars and a number of trading securities S . When $S > 0$, the fund holds this amount of securities, and when $S < 0$, it has borrowed this amount from other market participants to create a short position. In order to guarantee that the short-selling fund can return the borrowed securities to the lender at a later time, the fund sets aside a margin amount M equal to the current market value of the borrowings, in the form of cash. Fund managers may decide to borrow cash L up to a certain multiple of fund equity. For simplicity, only one interest rate applies to cash holdings, loans, and margin. This interest rate is the same as the interest rate obtained from holding the risk-free bond.

Table 4. This table details the balance sheet items used by all funds. All securities use the most recent market value in valuation.

Assets		Liabilities	
cash	C	Equity	
		capital	K
margin	M	Debt	
		loans	L
trading securities	S^+	borrowed securities	S^-

Table 5. This table provides a listing of the model parameters and their values.

parameter	value	description
g	0.02	dividend growth rate
k	0.02	cost of equity
σ	$6\% \sqrt{\frac{1}{252}}$	dividend growth volatility
θ	1	dividend autocorrelation lag
ω	0.1	dividend autocorrelation coefficient
ρ	$1 - 6^{-252} \sqrt{\frac{1}{2}}$	noise trader mean reversion rate
σ^{NT}	$12\% \sqrt{\frac{1}{252}}$	noise trader volatility
$\lambda_{NT}^*, \lambda_{VI}^*, \lambda_{TF}^*$	1, 8, 1	leverage limit
c_{NT}, c_{VI}, c_{TF}	5, 10, 4	signal scale

Wealth is calculated as:

$$W(t) = C(t) + S(t)p(t) - L(t) \quad [13]$$

The margin entry $M(t)$ on the balance sheet does not occur in this equation, as the margin account covers the negative part of $S(t)p(t)$ by holding its market value in cash. The funds can use *leverage*, meaning using borrowed funds to purchase additional risky assets. Leverage is a tool commonly used by fund managers, with a cursory look at public regulatory filings of U.S. institutional fund managers showing leverage ratios between 1 and 10*.

The investment mandate defines the fund managers' leverage constraints, which may be set by the external creditors, who are providing the fund with the needed loans, or internally – as a form of trading risk management. Given the leverage constraint $\lambda^* \in \mathbb{R}^+$, we can compute the maximum and minimum demand, in terms of the number of assets. Because we can have short positions, this set of portfolios is more general than the *budget set* as it also allows for negative amounts of the stock. The wealth of the fund develops as:

$$W(t+1) = W(t) + r[C(t) - L(t)] + [p(t+1) - p(t) + D(t)]S(t) \quad [14]$$

The leverage constraint is an integral part of the excess demand function. A fund can only violate its leverage constraint when the proportion of risky assets changes faster than the amount of equity capital. This can happen due to losses, or in rare cases when the market fails to clear completely. In those cases, the fund has the opportunity to reduce its risky position during the period via the inclusion of the leverage limit in the excess demand function. There still may be losses that exceed the fund's equity, making the net worth of the fund less than zero, and we require that all funds meet the solvency condition $W(t) > 0$. The simulation ends when one or more funds are insolvent. The model parameters, particularly the leverage constraint λ^* , influence the observed dynamics in the model. Table 5 lists the parameters used for the analysis in this paper.

Market Clearing. Prices are set by a price setter who chooses prices such that demand and supply match as close as possible. The excess demand of agent a for property i is defined in equation 2. The market excess demand curve for one particular investment i is the aggregate of the excess demand of all agents. As in the classical Walrasian setting, the price setter seeks to match

demand and supply, so that aggregate excess demand is zero for each investment, by finding a root of the market excess demand curve.

However, if no solution is found through the root-finding process, we must fall back to a heuristic that seeks for the best solution that only partially clears the market. We interpret the goodness of a solution as the extent to which the solution minimizes demand and supply *mismatch*. We here use the square of excess demand, and this way the root-finding problem is transformed into the corresponding minimization problem:

$$\begin{aligned} & \text{minimize}_p \quad \left(\sum_{a \in \mathcal{A}} E_a(p) \right)^2 \\ & \text{subject to} \quad \text{finite } p. \end{aligned}$$

Model and Software. The simulation in this paper builds on the *Economic Simulation Library*, an open-source library for agent-based modeling which is accessible at <https://github.com/INET-Complexity/ESL>.

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- DM Cutler, JM Poterba, LH Summers, What moves stock prices? *The J. Portfolio Manag.* **15**, 4–12 (1989).
- RJ Shiller, *Market volatility*. (MIT Press), p. 463 (1989).
- JD Farmer, Market force, ecology and evolution. *Ind. Corp. Chang.* **11**, 895–953 (2002).
- B LeBaron, Building the Santa Fe artificial stock market, (Brandeis University), Technical report (2002).
- A Lo, The adaptive market hypothesis: market efficiency from an evolutionary perspective. *The J. Portfolio Manag.* **30**, 15–29 (2004).
- T Hens, KR Schenk-Hoppe, *Handbook of financial markets : dynamics and evolution*. (North Holland), p. 584 (2009).
- JD Farmer, S Skouras, An ecological perspective on the future of computer trading. *Quant. Finance* **13**, 325–346 (2013).
- SJ Grossman, JE Stiglitz, On the Impossibility of Informationally Efficient Markets, Technical Report 3 (1980).
- N Beale, R Guntton, K L Bashe, H S Battey, R S MacKay, Dynamics of Value-Tracking in Financial Markets. *Royal Soc. Open Sci.* **11**, 1–22 (2019).
- B LeBaron, Empirical Regularities From Interacting Long-and Short-Memory Investors in an Agent-Based Stock Market, Technical Report 5 (2001).
- EF Fama, KR French, Dividend yields and expected stock returns. *J. Financial Econ.* **22**, 3–25 (1988).
- MJ Gordon, E Shapiro, Capital Equipment Analysis: The Required Rate of Profit (1956).
- B LeBaron, Agent-based computational finance in *Handbook of computational economics*, eds. L Tesfatsion, KL Judd. (Elsevier) Vol. 2, 1 edition, pp. 1187–1233 (2006).
- F Black, M Scholes, The effects of dividend yield and dividend policy on common stock prices and returns. *J. Financial Econ.* **1**, 1–22 (1974).
- WA Brock, CH Hommes, A Rational Route to Randomness. *Econometrica* **65**, 1059 (1997).
- WA Brock, CH Hommes, Heterogeneous beliefs and routes to chaos in a simple asset pricing model. *J. Econ. Dyn. Control.* **22**, 1235–1274 (1998).
- JD Farmer, S Joshi, The price dynamics of common trading strategies. *J. Econ. Behav. Organ.* **49**, 149–171 (2002).
- JD Farmer, Market force, ecology and evolution. *Ind. Corp. Chang.* **11**, 895–953 (2002).
- CS Asness, et al., Value and Momentum Everywhere. *The J. Finance* **LXVIII** (2013).
- JPP Bouchaud, et al., Black Was Right: Price Is Within a Factor 2 of Value, Technical report (2017).
- D Cherkashin, JD Farmer, S Lloyd, The reality game. *J. Econ. Dyn. Control.* **33**, 1091–1105 (2009).
- RM May, Will a large complex system be stable? *Nature* **238**, 413–414 (1972).
- M Novak, et al., Characterizing Species Interactions to Understand Press Perturbations: What Is the Community Matrix? *Annu. Rev. Ecol. Evol. Syst.* **47**, 409–432 (2016).
- JL Bronstein, Conditional outcomes in mutualistic interactions. *Trends Ecol. Evol.* **9**, 214–217 (1994).
- WB Arthur, WB Arthur, Asset Pricing Under Endogenous Expectations in an Artificial Stock Market in *The Economy as an Evolving Complex System II*. (CRC Press), pp. 15–44 (1997).
- R Franke, F Westerhoff, Structural stochastic volatility in asset pricing dynamics: Estimation and model contest. *J. Econ. Dyn. Control.* **36**, 1193–1211 (2012).
- S Thurner, JD Farmer, J Geanakoplos, Leverage Causes Fat Tails and Clustered Volatility (2011).
- A Kirilenko, AS Kyle, M Samadi, T Tuzun, The Flash Crash: High-Frequency Trading in an Electronic Market. *The J. Finance* **72**, 967–998 (2017).
- F Musciotto, L Marotta, J Piilo, RN Mantegna, Long-term ecology of investors in a financial market. *Palgrave Commun.* **4**, 92 (2018).

* The Electronic Data Gathering, Analysis, and Retrieval system (EDGAR) <https://www.sec.gov/edgar.shtml>. Securities and Exchange Commission