Targeted Carbon Tax Reforms*

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Abstract

We show that in the presence of intersectoral linkages targeted sectoral carbon taxes might be a more effective way of reducing emissions than economy-wide carbon pricing. A carbon tax imposed on *all* sectors unambiguously reduces aggregate emissions, but taxes targeted at the set of *key sectors* can lead to the greatest emissions reduction. Due to intersectoral linkages and a tax rebate effect, taxing non-key sectors dampens the reduction in aggregate emissions. A key sector typically not only produces a lot of emissions, but also has a large influence on emissions in the rest of the economy. We focus on incremental changes in carbon taxes—carbon tax *reforms*—to characterise the set of key sectors analytically.

Keywords: emissions tax, carbon tax, pollution tax, climate change, environmental tax reform, second-best policy, input-output linkages, intersectoral network, cap-and-trade.

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1 Introduction

To have a reasonable chance of avoiding the dangerous effects of climate change, cumulative anthropogenic carbon emissions must be limited to one trillion tonnes (Allen et al., 2009, Meinshausen et al., 2009). One of the key questions in the economic analysis of climate change policies is: what is the most effective way of achieving this target?[1] In this paper, we show that, due to intersectoral linkages, taxation of certain *key sectors* might be a more effective way of reducing emissions than economy-wide pricing. The reason is that the tax rebate effect from taxing a sector might exceed the sector's aggregate, network-adjusted impact on aggregate emissions reductions. Taxation of such (non-key) sectors would dampen the effectiveness of carbon taxes. To derive our results analytically, we focus on incremental (or marginal) sectoral carbon tax changes, which we refer to as *carbon tax reforms*.[2]

We analyse the effects of carbon tax reforms in an *intersectoral input-output network* in which firms trade intermediate inputs and sell their goods to a representative consumer. A sector's use of different intermediate inputs produces emissions directly and indirectly. For example, a factory might use coal, electricity, and labour. For every unit of coal that it uses, the factory produces emissions directly. But when the factory uses electricity, it does not produce emissions directly; rather the utility company that supplies the factory is likely to produce emissions in the process of electricity generation. Therefore, by using electricity the factory also produces emissions indirectly. We assume that the government can impose *sectoral* carbon taxes and that it rebates the tax revenue in full to the consumer. The government's objective is to implement sectoral carbon taxes that reduce aggregate emissions by the greatest amount.

We analytically characterise the *most effective* carbon tax reform in the presence of intersectoral linkages. Our main insight is that the most effective carbon tax reform might not involve taxing all sectors. We show that the effect of a tax reform on aggregate emissions

[1] The calculation of optimal carbon taxes is usually done with sophisticated Integrated Assessment Models (IAMs) that combine a {multi-sectoral, multi-regional, dynamic, stochastic, computable} general equilibrium model of the economy and an earth science model (Metz et al., 2001). These models are broadly of two types. Policy evaluation IAMs, such as GCAM, IMAGE, and MESSAGE, used by the Intergovernmental Panel on Climate Change, calculate the most cost-effective paths to a fixed emissions target (e.g. net zero carbon emissions by 2050). Policy optimisation IAMs, such as DICE/RICE, PAGE, and FUND additionally include a damage function (that maps emissions to the effects on output) which allows them to calculate the socially optimal level of cumulative emissions. Rather than designing a socially optimal or a cost-effective carbon tax policy, we focus on the direction of policy changes that achieves the biggest reduction in emissions on the margin.

[2] Thus our analytical results are most likely to be relevant for cases in which the government faces political economy constraints, such as lobbying, that limit the size of tax changes that it can feasibly negotiate. Our numerical results suggest, however, that a targeted approach might also be more effective at reducing emissions even when the government can impose larger carbon taxes.

can be separated into the contributions of individual sectors, but due to intersectoral linkages these can be positive or negative for any given sector. A sector is a key sector if taxing its emissions has a negative impact on aggregate emissions. Typically, key sectors are not only very polluting, but also have a substantial influence on the emissions of other sectors via intersectoral linkages.

A sectoral carbon tax affects aggregate emissions in three ways. First, a taxed sector's production mix switches from direct inputs that are polluting to cleaner ones. The fall in input demands reduces the production and emissions of all the indirect suppliers to the sector. We call this the sector's *upstream emissions influence*. Second, the taxed sector's output price rises which causes its buyers to switch away from using the sector's good as an input and reduces the output and emissions of all direct and indirect buyers. We call this the sector's *downstream emissions influence*. We refer to the sum of upstream and downstream influences as the sector's *aggregate emissions influence*. Third, the tax revenue is rebated to consumer who is therefore able to consume more from *every* sector. This *tax rebate effect* increases sectoral output and aggregate emissions.

Whether taxing a sector has a positive or negative impact on aggregate emissions depends on (i) the tax rebate effect, (ii) the sector's aggregate emissions influence, and (iii) the sector's level of emissions relative to aggregate emissions. If the sector's emissions are sufficiently high, the sector will always be a key sector. But even sectors with low emissions might have high aggregate emissions influence in the intersectoral network, and could hence be key sectors. However, if sectoral emissions are low, then the tax rebate effect can exceed the other effects, netting a positive effect on aggregate emissions.[3]

Our paper connects two strands of the literature: the economics of intersectoral input-output networks and of environmental tax reforms. The first strand is a rapidly growing literature that looks at the propagation of shocks in an interconnected economy. In the zero-tax benchmark, our general equilibrium model is a static version of Long Jr. and Plosser's (1983) real business cycles model which is analysed by Acemoglu et al. (2012). In this paper, we analyse the effects of adding emissions and emissions taxes to that model. Sectors produce emissions according to an emissions function which is assumed have constant returns to scale in the inputs to production and we place no restrictions on the structure of the input-output network. The upstream and downstream effects of carbon tax reforms in our model are similar to the structure of technology shock propagation in other models (Shea, 2002,

[3] The possibility of a positive effect on aggregate emissions resulting from sectoral taxes in our model is very different from the channels described in carbon leakage (i.e. offshoring of production due to domestic emission pricing; see, for example, Babiker, 2005). The tax rebate to the consumer is central to our results, but it plays little role in carbon leakage. Moreover, carbon leakage models allow firms to relocate their production to a different country whereas our model does not.

Baqaee, 2015, Huremovic and Vega-Redondo, 2016).[4] In an important contribution, Baqaee (2016) showed that in the presence of intersectoral linkages fiscal policy could be targeted at certain sectors in order to maximise impact on employment and output; our results have a similar flavour. Our set-up is also related to models used in the input-output analysis of carbon content of consumption and production (e.g. Turner et al., 2007, Wiedmann et al., 2007, Wiedmann, 2009, Davis and Caldeira, 2010, Caron et al., 2016).

The second strand is a rich literature on environmental tax reforms. This work stems from the classic public economics debate on optimal tax design vs. welfare-improving tax reform (Buchanan, 1976, Feldstein, 1976, Guesnerie, 1977, Weymark, 1981).[5] One of the key questions in the environmental tax reforms literature is whether the shift of the tax burden away from employment and income towards consumer-harming pollution makes the consumer better off (Copeland, 1994, Bovenberg and De Mooij, 1994, Bovenberg and van der Ploeg, 1994, Bovenberg and Goulder, 1996, Bovenberg and van der Ploeg, 1996).[6] In contrast, in our model, emissions do not affect utility directly i.e. emissions are not an externality. The government simply has an exogenous reason to reduce carbon emissions (e.g. adhering to the Nationally Determined Contribution as part of the Paris Agreement). Moreover, we assume that all the tax revenue is rebated in full to the consumer and that the economy has no distortions (such as income or commodity taxes) or unemployment prior to the introduction of a carbon tax reform. Utility is maximised under this status quo which implies that (by the envelope theorem) any new *incremental* taxes come at no cost to utility.

This paper is organised as follows. Section 2 introduces the model. Section 3 provides the benchmark solution in the absence of emissions taxation. Section 4 considers the effect of carbon tax reforms on sectoral consumption, labour demand, intermediate input use, output, and emissions. Section 5 identifies the key sectors and characterises the most effective carbon tax reform. Section 6 presents a numerical illustration of the main results. Section 7 discusses possible extensions of the model. Section 8 concludes. The appendix provides the proofs and further results of the numerical illustration.

[4]Since taxes introduce non-linearities into our model, our tax reform approach uses the first-order approximation approach suggested by Acemoglu et al. (2015).

[5]See Myles (1995, Chapter 6) for a summary.

[6]See Bovenberg and Goulder (2002, Section 3) for a summary.

2 Model

2.1 Sectors

Our model and notation build directly on Acemoglu et al. (2012). Each of n competitive sectors produces a distinct good. Sector i produces output Y_i according to the Cobb-Douglas production function

$$Y_i = l_i^{1-\alpha} \prod_{j=1}^{n} x_{ij}^{\alpha w_{ij}}, \tag{2.1}$$

where $w_{ij} \geq 0$ is the share of expenditure on input j in sector i's expenditure on intermediate inputs, $\alpha \in (0, 1)$ is the share of intermediate goods in production, x_{ij} is the input demand of sector i for the good produced by sector j, and l_i is sector i's demand for labour.[7] The matrix $W = [w_{ij}]$ is the economy's *input-output* matrix. We assume that $\sum_{j=1}^{n} w_{ij} = 1$ for all i which ensures constant returns to scale in production.

The emissions of sector i are determined by a constant returns to scale emissions function $E_i(l_i, x_{i1}, ..., x_{in})$. Sectoral emissions are a function of inputs directly rather than a function of sectoral output. Therefore, depending on its input mix, a sector can produce the same level of output and different levels of emissions. Each sector i is subject to a per-unit tax t on its emissions. The profit of (a representative firm in) sector i is therefore given by

$$\pi_i = p_i Y_i - \sum_{j=1}^{n} p_j x_{ij} - \omega l_i - \lambda_i t E_i, \tag{2.2}$$

where ω is the competitive wage rate and $\lambda_i \in \{0, 1\}$ is a parameter. A sector is i taxed if and only if $\lambda_i = 1$. By selecting an appropriate vector $\lambda \in \{0, 1\}^n$ this set-up allows us to analyse the effect of emissions taxes on any subset of sectors.

2.2 Consumer

A representative consumer inelastically supplies one unit of labour l that can be hired by the sectors, and has Cobb-Douglas preferences over consumption[8]

$$U(C_1, ..., C_n) = \prod_{i=1}^{n} C_i^{1/n}. \tag{2.3}$$

[7]Since our focus is not on total factor productivity shocks, we normalise the usual TFP parameter to 1.
[8]The results in the paper are unaffected if we set $U(C_1, ..., C_n) = \prod_{i=1}^{n} C_i^{\gamma_i}$ with $\gamma_i \geq 0$.

The utility function does not depend on the level of emissions. We take the government's reasons to reduce emissions as given and we are not attempting to estimate the *socially optimal level* of emissions. Emissions are not an externality in this model but rather act as a friction on the production side in the presence of emissions taxes.

We assume perfect mobility of labour across sectors so there is a unique competitive wage ω. We set labour to be the numéraire good. The government redistributes the emissions tax revenue in full to the consumer. Therefore, the consumer's budget constraint is given by

$$\sum_{i=1}^{n} p_i C_i \leq \omega l + T = 1 + T, \tag{2.4}$$

where $T = t \sum_{i=1}^{n} \lambda_i E_i$ is the total emissions tax revenue. There are no profits from sector ownership due to the constant returns to scale exhibited by production and emissions.

3 Zero-tax benchmark

When the emissions tax is zero, our economy essentially coincides with the economy presented in Acemoglu et al. (2012).

Definition 1. A *competitive equilibrium* consists of prices $p_1, \ldots, p_n$, a wage ω, consumption levels $C_1, \ldots, C_n$, labour demands $l_1, \ldots, l_n$, and intermediate input quantities x_{ij} for all i, j, such that (i) the consumer maximises her utility subject to her budget constraint, (ii) the sectors maximise their profits, (iii) the markets for each good and labour clear; that is,

$$\sum_{i=1}^{n} l_i = l = 1, \tag{3.1}$$

and, for each i,

$$Y_i = C_i + \sum_{j=1}^{n} x_{ji}. \tag{3.2}$$

In order to characterise the competitive equilibrium, we will employ standard definitions from the networks literature (Bonacich, 1987, Ballester et al., 2006, Jackson, 2008).

Definition 2. The matrix $V = [v_{ij}] = (I - \alpha W')^{-1}$ is the economy's *Bonacich matrix*.[9]

We can alternatively write the Bonacich matrix as

$$V = I + \alpha(W') + \alpha^2 (W')^2 + \ldots \tag{3.3}$$

[9] Acemoglu et al. (2012) refer to V as the *Leontief inverse* while Acemoglu et al. (2015) refer to the transposed matrix V' as the *Leontief matrix*.

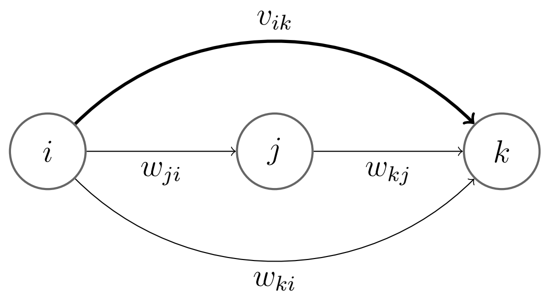

Figure 3.1: Thin arrow $i \xrightarrow{w_{ki}} k$ represents how much of sector k's production depends *directly* on good i. Thick arrow $i \xrightarrow{v_{ik}} k$ represents how much sector k's production depends *directly and indirectly* (via j) on good i.

The ik element of the matrix W', w_{ki}, measures how much of sector k's production depends directly on the use of the good produced by sector i. In Figure 3.1 we represent this as a thin arrow going from i to k (only links with positive weight are shown). The ik element of the matrix $(W')^2$ aggregates all weighted walks of length two going from i to k: it measures how reliant k's production is on the use of good i through its use of the intermediate input j. In the network shown in Figure 3.1, the ik element of $(W')^2$ is equal to the product $w_{kj}w_{ji}$. Similar reasoning applies to the elements of $(W')^h$ for $h > 2$. The element v_{ik} then measures how reliant sector k's production is on the output of sector i taking *all direct and indirect* effects into account. We therefore refer to the term as the *downstream influence* of i on k. We will also interchangeably refer to this term as the *upstream influence* of k on i since v_{ik} also measures how reliant sector i's production is on input demand by sector k taking all direct and indirect effects into account. In Figure 3.1, $v_{ik} = \alpha w_{ki} + \alpha^2 w_{kj} w_{ji}$ is represented by a thick arrow going from i to k.

The Bonacich centrality of sector i is its average downstream influence on others or, equivalently, the average upstream influence of other sectors on i. In other words, it measures direct and indirect reliance of other sectors on sector i's output.

Definition 3. The *Bonacich centrality* of sector i is given by $b_i = \frac{1}{n}\sum_{j=1}^{n} v_{ij}$.

Proposition 1 shows that b_i is precisely the value of the sales of sector i in the zero-tax equilibrium (Hulten, 1978).

Proposition 1 (Acemoglu et al., 2012)**.** When $t = 0$ there is a unique competitive equilibrium and it is characterised by

$$p_i^* = \exp\left(-A\sum_{j=1}^{n} v_{ji} - \alpha\sum_{j=1}^{n} v_{ji}\sum_{h=1}^{n} w_{jh}\ln(w_{jh})\right), \tag{3.4}$$

$$C_i^* = 1/(np_i^*), \tag{3.5}$$

$$s_i^* = b_i, \tag{3.6}$$

$$Y_i^* = s_i^*/p_i^*, \tag{3.7}$$

$$x_{ij}^* = s_i^*\alpha w_{ij}/p_j^*, \tag{3.8}$$

$$l_i^* = s_i^*(1-\alpha), \tag{3.9}$$

where $A = (1-\alpha)\ln(1-\alpha) + \alpha\ln(\alpha)$, and the sales of sector i are defined by $s_i = p_i Y_i$. The equilibrium emissions of sector i when $t = 0$ are given by $E_i^* = E_i(l_i^*, x_{i1}^*, ..., x_{in}^*)$.

4 Carbon tax reforms

We now consider the impact of introducing a carbon tax reform around our benchmark zero-tax equilibrium.[10]

Definition 4. For any sector i, *input j's elasticity of emissions* evaluated at $t = 0$ is

$$\epsilon_{ij} = \frac{x_{ij}^*}{E_i^*}\frac{\partial E_i}{\partial x_{ij}}\bigg|_{t=0}. \tag{4.1}$$

For each i and j we assume that $\epsilon_{ij} > 0$ only if $w_{ij} > 0$. We refer to $\mathcal{E} = [\epsilon_{ij}]$ as the *emissions elasticity* matrix or as the *emissions network*. We can also define $\epsilon_{il} = \frac{l_i^*}{E_i^*}\frac{\partial E_i}{\partial l_i}\big|_{t=0}$ to be the labour elasticity of emissions for sector i evaluated at $t = 0$, but for the clarity of exposition we set $\epsilon_{il} = 0$ for all i. Emissions elasticities will be crucial in determining how emissions taxes propagate through intersectoral linkages.

Definition 5. The *downstream emissions influence* of sector i on sector k or, equivalently, the *upstream emissions influence* of sector k on sector i is

$$\psi_{ik} = \sum_{j=1}^{n} v_{ij}\epsilon_{kj}. \tag{4.2}$$

[10] When emissions taxes are strictly positive, the system of equations that determines competitive equilibria in our economy becomes highly non-linear. Therefore, we cannot give a closed-form analytical solution to the effects of a tax reform around an arbitrary positive emissions tax.

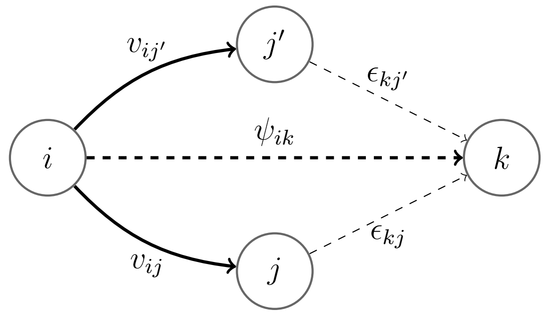

Figure 4.1: Thin dashed arrow $j \xrightarrow{\epsilon_{kj}} k$ represents input j's elasticity of emissions for sector k. Thick dashed arrow $i \xrightarrow{\psi_{ik}} k$ represents how much a change in k's demand for inputs affects sector i following a tax on sector k's emissions.

Note that ψ_{ik} is the ik element of the matrix $\Psi = [\psi_{ij}] = V\mathcal{E}'$.

Let us now illustrate ψ_{ik} as upstream emissions influence of k on i (Figure 4.1).[11] Recall that ϵ_{kj} measures how sensitive the emissions of sector k are to the use of input j. Following a tax on its emissions, sector k reduces its demand for input j.[12] Since demand for good j has fallen, sector j reduces its demand for its direct (and indirect) inputs. Recall that v_{ij} measures the effect of j's input demand reduction on sector i (i.e. upstream influence of j on i). Aggregating over all of k's direct inputs (i.e. j and j' in Figure 4.1), ψ_{ik} measures how an emissions tax on sector k would affect the demand for sector i's good via the changes in demand for all of k's direct and indirect inputs.

Armed with precise interpretations of the ψ_{ik} and v_{ik} terms, we are now in a position to trace the effects of carbon tax reforms on relative prices and on real variables (intermediate input use, labour demand, sectoral output, sectoral emissions, and consumption).[13]

[11]In general, sector i could be *both* upstream and downstream from sector k. In that case, sector i will be subject to both upstream emissions influence ψ_{ik} and downstream emissions influence ψ_{ki} of k on i. In our analysis of tax reforms these effects turn out to be completely separable.

[12]Sector k reduces its demand for all inputs, but demand falls relatively more for more polluting inputs.

[13]Utility is a function only of consumption and is maximised in the zero-tax equilibrium (i.e. $\frac{\partial U}{\partial C_i}\big|_{t=0} = 0$ for all i). Therefore, by the envelope theorem, the effect of incremental taxation on the consumer's utility is zero. In particular,

$$\frac{\partial U}{\partial t}\bigg|_{t=0} = \sum_{i=1}^{n} \frac{\partial U}{\partial C_i}\bigg|_{t=0} \cdot \frac{\partial C_i}{\partial t}\bigg|_{t=0} = 0.$$

Proposition 2. Consider any sectoral tax vector $\lambda \in \{0,1\}^n$, then[14]

$$\left.\frac{\partial \ln(p_i)}{\partial t}\right|_{t=0} = \sum_{h=1}^{n} \lambda_h E_h^* \frac{\upsilon_{hi}}{b_h}, \tag{4.3}$$

$$\left.\frac{\partial \ln(x_{ij})}{\partial t}\right|_{t=0} = \sum_{h=1}^{n} \lambda_h E_h^* \left(1 - \frac{\psi_{ih}}{b_i} - \frac{\upsilon_{hj}}{b_h}\right) - \lambda_i \frac{E_i^*}{b_i}\frac{\epsilon_{ij}}{\alpha w_{ij}}, \tag{4.4}$$

$$\left.\frac{\partial \ln(l_i)}{\partial t}\right|_{t=0} = \sum_{h=1}^{n} \lambda_h E_h^* \left(1 - \frac{\psi_{ih}}{b_i}\right), \tag{4.5}$$

$$\left.\frac{\partial \ln(Y_i)}{\partial t}\right|_{t=0} = (1-\alpha)\left.\frac{\partial \ln(l_i)}{\partial t}\right|_{t=0} + \alpha \sum_{j=1}^{n} w_{ij} \left.\frac{\partial \ln(x_{ij})}{\partial t}\right|_{t=0}, \tag{4.6}$$

$$\left.\frac{\partial \ln(E_i)}{\partial t}\right|_{t=0} = \sum_{j=1}^{n} \epsilon_{ij} \left.\frac{\partial \ln(x_{ij})}{\partial t}\right|_{t=0}, \tag{4.7}$$

$$\left.\frac{\partial \ln(C_i)}{\partial t}\right|_{t=0} = \sum_{h=1}^{n} \lambda_h E_h^* \left(1 - \frac{\upsilon_{hi}}{b_h}\right). \tag{4.8}$$

We can unpack the contents of Proposition 2 by considering a special case in which only sector k's emissions are taxed (so $\lambda_k = 1$ and $\lambda_i = 0$ for all $i \neq k$). Let us consider the effect on prices first. Equation (4.3) shows that, when sector k's emissions are taxed, only the prices of goods produced by sectors downstream from k increase. The sectors downstream from k face a new set of relative input prices, which distorts their input mix, thereby reducing their output and increasing their prices. However, the first-order conditions that determine the input choices of sectors upstream from k do not change: upstream sectors simply face a new levels of demand by sector k (directly or indirectly) for their output. In particular, less polluting upstream sectors face relatively more demand from k and more polluting inputs face relatively less demand. In response to the new demands for the goods, sectors upstream from k simply scale the use of *all* their inputs thereby maintaining the same prices and zero profits for all upstream sectors.

Equation (4.4) states that following the incremental tax on k's emissions, the change in sector i's demand for good j (where $i \neq j \neq k$) is

$$E_k^* \left(\underbrace{1}_{\substack{\text{tax rebate}\\\text{effect}}} \underbrace{-\frac{\psi_{ik}}{b_i}}_{\substack{\text{relative upstream}\\\text{emissions influence}}} \underbrace{-\frac{\upsilon_{kj}}{b_k}}_{\substack{\text{relative downstream}\\\text{influence}}} \right). \tag{4.9}$$

The term E_k^* outside the bracket states the obvious: the higher the emissions of sector k,

[14] Strictly speaking, the derivative of input j with respect to tax for sector i should be set to zero when $w_{ij} = 0$.

the greater its impact on the whole network. The first term inside the brackets is the tax rebate effect: since the consumer has a higher income, she can consume more of good i. This increases sector i's demand for all its inputs. The second term is the *relative* upstream emissions influence of sector k on sector i. The reduction in sector k's demand for inputs dampens sector i's demand for inputs. However, the magnitude of this effect is attenuated by the sales (Bonacich centrality) of i. The third term is the *relative* downstream influence of sector k on sector j. When sector k is taxed, its output price rises. This raises the price of downstream sector j's good which, in turn, reduces i's demand for good j. The magnitude of this effect is attenuated by the sales of sector k since taxing the emissions of a sector with large sales (relative to a fixed level of emissions E_k^*) will have a small impact on its price and therefore on the prices of its downstream sectors.

To understand the relevance of the final term in equation (4.4) we need to consider a setting in which sector i's emissions are taxed (so $\lambda_i = 1$). This term is always negative and measures the *additional distortion* to the first-order conditions of sector i from a tax on its own emissions. The magnitude of the distortion on sector i's input mix is greater when (i) emissions are high relative to sales (E_i^*/b_i) and (ii) when input j is more polluting relative to its expenditure share ($\epsilon_{ij}/\alpha w_{ij}$).

Let us now look at the effects of a tax reform on the remaining variables in the case in which only sector k's emissions are taxed. The labour demand of sector i is affected only by the tax rebate effect and by the relative upstream emissions influence of k on i (see equation 4.5). Since labour is taken to be the numéraire good before and after the tax, the wage remains unchanged, and therefore there is no downstream (or price) effect of an emissions tax on labour demand.[15] The impact of the tax reform on sectoral output and sectoral emissions can be decomposed into weighted sums of changes in the use of the relevant inputs (see equations 4.6 and 4.7). In the case of output, the weights are simply expenditure shares on factors of production, and, in the case of emissions, the weights are emissions elasticities. Finally, from equation (4.8), we can observe that the change in the consumption of good i depends only on the tax rebate effect and on the relative downstream influence of sector k on sector i. This is because the consumer cares only about relative prices of goods and only the prices of goods produced by sectors downstream from k change following an emissions tax on k.

[15] Furthermore, since we assumed that the labour elasticity of emissions is zero, there is no additional distortionary effect on labour from an emissions tax on sector i.

5 Targeted carbon tax reforms

In this section we analyse the impact of carbon tax reforms on *aggregate* emissions. The *most effective* carbon tax reforms are the sets of incremental sectoral taxes that produce the greatest reduction in aggregate emissions. We can state this more formally.

Definition 6. The *most effective* carbon tax reforms are vectors λ^* which satisfy

$$\lambda^* \in \arg\min_{\lambda \in \{0,1\}^n} \left\{ \left. \frac{\partial \ln(E)}{\partial t} \right|_{t=0} \text{ such that } \left. \frac{\partial \ln(E)}{\partial t} \right|_{t=0} < 0 \right\}. \tag{5.1}$$

In order to characterise the most effective carbon tax reforms, we first describe the contribution of each taxed sector to the change in aggregate emissions following any carbon tax reform. Let e_i denote the ith standard basis vector.

Proposition 3. For any sectoral tax vector $\lambda \in \{0,1\}^n$,

$$\left. \frac{\partial \ln(E)}{\partial t} \right|_{t=0} = \sum_{i=1}^{n} \lambda_i \left. \frac{\partial \ln(E)}{\partial t} \right|_{\substack{\lambda = e_i \\ t=0}} \tag{5.2}$$

where

$$\left. \frac{\partial \ln(E)}{\partial t} \right|_{\substack{\lambda = e_i \\ t=0}} = E_i^* - \sum_{j=1}^{n} \frac{E_i^* E_j^*}{E^*} \left(\frac{\psi_{ji}}{b_j} + \frac{\psi_{ij}}{b_i} \right) - \frac{(E_i^*)^2}{E^*} \left(\sum_{j=1}^{n} \frac{\epsilon_{ij}^2}{\alpha w_{ij} b_i} \right). \tag{5.3}$$

Equation (5.2) states that the overall change in aggregate emissions following a carbon tax reform can be additively decomposed into the effects of taxes on individual sectors. We call the effect on aggregate emissions of taxing sector i alone (equation 5.3) the *aggregate emissions impact* of sector i. So the effect (on aggregate emissions) of taxing the emissions of sectors i and k is equal to the aggregate emissions impact of sector i plus the aggregate emissions impact of sector k. Importantly, due to the presence of intersectoral linkages, the aggregate emissions impact of a sector i is different from the change in the emissions of sector i following a carbon tax on i alone (that is, $\left. \frac{\partial \ln(E)}{\partial t} \right|_{\substack{\lambda = e_i \\ t=0}} \neq \left. \frac{\partial \ln(E_i)}{\partial t} \right|_{\substack{\lambda = e_i \\ t=0}}$).

Let us now analyse the aggregate emissions impact of sector i i.e. the terms of equation (5.3). The first term is simply the tax rebate effect. The higher the emissions of sector i, the greater the amount of tax revenue that the government can raise following a tax. As before, this effect makes the consumer wealthier which induces greater spending across the economy, higher output, and higher emissions. This is the only positive term in the expression.[16] We call the second term sector i's *aggregate emissions influence*: it is a weighted sum of relative

[16]In a partial equilibrium setting in which only a fraction of the tax is rebated to the consumer, this term would be scaled down correspondingly.

upstream and relative downstream emissions influences. Note that the relative downstream emissions influence part is new: when i is taxed, i's price rises which distorts the input mix of its downstream sectors, thereby reducing their output and their emissions. The final term is the familiar additional distortion to sector i when its own emissions are taxed (cf. equation 4.4).

It should be clear that if the size of the tax rebate effect outweighs the aggregate emissions influence and the additional distortion then the aggregate emissions impact of a sector will be positive. Nevertheless, taxing the emissions of *all* (emitting) sectors will unambiguously reduce aggregate emissions as the following proposition shows.

Proposition 4. If $\lambda = \mathbf{1}$, then

$$\left.\frac{\partial \ln(E)}{\partial t}\right|_{t=0} \leq 0. \tag{5.4}$$

However, because some sectors may have a positive aggregate emissions impact, taxing all sectors might not constitute the most effective carbon tax reform. We therefore focus on sectors whose aggregate emissions impact is negative.

Definition 7. Sector i is a *key sector* if its aggregate emissions impact is negative.

Using equation (5.3), we can see that a sector is more likely to be a key sector if it has a large aggregate emissions influence or high emissions. Indeed, the aggregate emissions impact of sector i is inverse-U-shaped in sector i's emissions. So, for low levels of sector i's emissions, the tax rebate term might outweigh the additional distortion term (potentially rendering the aggregate emissions impact positive), whereas when sector i's emissions are sufficiently high, the (negative quadratic) additional distortion term will swamp out the other two effects.

As Proposition 3 shows, the impact of a multi-sector carbon tax reform on aggregate emissions is the sum of the aggregate emissions impacts of each taxed sector. Therefore, taxing any key sector will reduce emissions while taxing any non-key sector will dampen this reduction. This insight allows us to fully characterise the most effective carbon tax reform.

Theorem. In the most effective carbon tax reform, sector i's emissions are taxed if and only if i is a key sector.[17]

[17] If the government were also able to *subsidise* certain sectors then the most effective carbon policy reform would be

$$\lambda^* \in \underset{\lambda \in \{-1,0,1\}^n}{\arg\min} \left\{ \left.\frac{\partial \ln(E)}{\partial t}\right|_{t=0} \text{ such that } \left.\frac{\partial \ln(E)}{\partial t}\right|_{t=0} < 0 \text{ and } T > -1 \right\}.$$

That is, the government can choose which sectors to tax and subsidise subject to the constraint that the consumer's income remains strictly positive. The solution to this problem is an immediate consequence of Proposition 3: tax all the key sectors and subsidise as many sectors with the largest positive aggregate emissions impacts as possible. The reason subsidies for non-key sectors reduce emissions in our model is that the

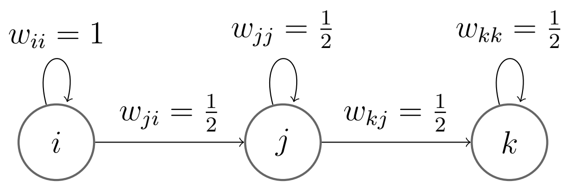

Figure 6.1: The economy in our numerical illustration. Only input-output links with strictly positive values are shown.

6 Numerical illustration

We now illustrate our main results with a numerical example. We set the labour share in production for each sector to 2/5 (i.e. $1-\alpha = 2/5$).[18] Our economy's input-output matrix is represented in Figure 6.1. We show only the input-output links which have strictly positive values. Finally, we specify a linear emissions function:

$$E_h = \sum_{l \in \{i,j,k\}} g_{hl} x_{hl}, \tag{6.1}$$

where $h \in \{i, j, k\}$, and $g_i = (20, 0, 0)$, $g_j = (1, 1, 0)$, and $g_k = (0, 1, 1)$ are chosen as parameter values. This function clearly exhibits constant returns to scale. In this example, sector i produces a lot of emissions when it uses its own good as an input while sectors j and k are not very polluting when they use any of their inputs.

The competitive equilibrium outcome for the zero-tax benchmark is reported in Table 1 (cf. Proposition 1). The final column ("Impact") shows the aggregate emissions impact of each sector, which measures the percentage change in aggregate emissions in response to a unit (or marginal) increase in the carbon tax on the emissions of that sector alone (cf. equation 5.3). Clearly, sectors i and j are key sectors (with i having the largest aggregate emissions impact) while sector k is not a key sector. The level of aggregate emissions in the economy is 3.0821.

We now present results from imposing carbon taxes on various subsets of sectors. We focus only on the most relevant changes from the zero-tax benchmark here (the full results tables are in Appendix D). In each case, the magnitude of the per-unit tax on emissions is set to 0.01. Let us start by taxing only sector j (see Table 2). Aggregate emissions fall to 3.0812. If we instead tax only sector k then, due to k's positive aggregate emissions impact, aggregate

reduction in the consumer's income (the negative tax rebate effect) exceeds the (positive) emissions influence and additional distortion terms. This policy reform relies heavily on the assumption of fixed technology: if firms can adjust their production functions, they could easily exploit the subsidy while increasing emissions.

[18]The labour share of income is around 50 percent in advanced economies and around 37 percent in emerging economies (IMF, 2017).

	Price	Output	Consumption	Emissions	Input i	Input j	Input k	Impact
i	5.3791	0.2498	0.0620	2.9972	0.1499	0.0000	0.0000	−11.5654
j	9.7440	0.0698	0.0342	0.0589	0.0379	0.0209	0.0000	−0.0315
k	12.5696	0.0379	0.0265	0.0260	0.0000	0.0147	0.0114	0.0087

Table 1: The zero-tax benchmark in our numerical illustration.

emissions rise to 3.0824 (see Table 3). Taxing *all* sectors reduces aggregate emissions to 2.7684 (see Table 4). However, the most effective carbon tax reform, which imposes carbon taxes only on the key sectors i and j, results in even lower emissions at 2.7682 (see Table 5). Note that the −10.2 percent change in emissions in the computed general equilibrium following the tax on every sector is *approximately* the sum of the theoretical aggregate emissions impacts (cf. equation 5.2 in Proposition 3).[19] While our *analytical* results are valid only for small taxes, we nevertheless found that targeting key sectors is also the most effective reform when we increased the magnitude of the sectoral taxes to 1 (holding all other parameter values in our numerical illustration fixed).

7 Discussion

There are several ways to enrich the production side of our economy. One could include firm profits (Baqaee, 2015, Huremovic and Vega-Redondo, 2016), financial frictions (Bigio and La'O, 2016) or other fundamental market distortions (Liu, 2017), firm entry and exit (Baqaee, 2015), dynamics and unemployment (Baqaee, 2016), distributional concerns (Klenert et al., 2016), international trade (Antweiler et al., 2001, Davis and Caldeira, 2010, Bosker and Westbrock, 2014),[20] and production network formation (Acemoglu and Azar, 2017, Oberfield, 2017).

Additionally, our model could be used to examine several other policy levers beyond the sector-specific emissions taxes that we have focussed on.

7.1 Cap-and-trade scheme

Instead of taxing sectors and redistributing revenue to the consumer, suppose that each sector i is initially allocated an emissions permit allowance $\nu_i Q$ where $\sum_{i=1}^{n} \nu_i = 1$. Here, Q is the overall emissions cap and ν_i is sector i's share of the cap. Emissions are competitively traded across firms at price p_Q. Then (a representative firm in) each sector i solves the

[19] The precision of the approximation worsens as the size of the tax increases.

[20] An international version of our model coupled with firm entry and exit would also allow us to investigate carbon leakage (Babiker, 2005).

following profit maximisation problem:

$$\max_{l_i, x_{ij}} \pi_i = \max_{l_i, x_{ij}} \; p_i Y_i - \sum_{j=1}^{n} p_j x_{ij} - l_i - p_Q E_i + p_Q \nu_i Q. \tag{7.1}$$

The market for emissions permits clears when $\sum_{i=1}^{n} E_i = Q$. The government can choose the aggregate cap Q and the sector-specific initial permit allocations ν_i.

We could examine two policy reforms of a cap-and-trade scheme. First, we could analyse $\frac{\partial p_Q}{\partial \nu_i}$ i.e. the effect of incrementally reducing one sector's initial permit allocation on the price of emission permits. Second, we could look at $\frac{\partial E_i}{\partial Q}$ to determine the distributional impact on emissions across the sectors of incrementally tightening the cap.[21]

7.2 Polluting input tax reform

Our model can also handle taxation of polluting inputs. For example, in the power sector the government not only observes emissions but is often able to estimate the emissions function. In this case, each sector i could be subjected to input taxes (subsidies) τ_{ij} that apply to the dirtiness of each input ($\frac{\partial E_i}{\partial x_{ij}} x_{ij}$). Now (a representative firm in) each sector i would solve the following profit maximisation problem:

$$\max_{l_i, x_{ij}} \pi_i = \max_{l_i, x_{ij}} \; p_i Y_i - \sum_{j=1}^{n} p_j x_{ij} - l_i - \sum_{j=1}^{n} \tau_{ij} \frac{\partial E_i}{\partial x_{ij}} x_{ij}. \tag{7.2}$$

Under this policy reform, rather than identifying the set of key sectors, we would be interested in the set of *key sector-specific inputs* i.e. the set of inputs which, when taxed, would produce the greatest reduction in aggregate emissions.

Of course, if we set $\frac{\partial E_i}{\partial x_{ij}} = 1$ for all i and j then we would formally model the case of general input taxation in an intersectoral network. Our model can therefore be used to analyse a wide set of tax policies. For example, in an international trade version of our model, input-specific taxes could allow us to trace the effects of border carbon adjustments (Fischer and Fox, 2012, Helm et al., 2012).

8 Conclusion

This paper formally analyses carbon taxes in the presence of intersectoral linkages. Our results highlight the importance of considering general equilibrium effects when implementing

[21] An analytically tractable analysis might begin with the status quo in which $Q = E^*$ and $\nu_i Q = E_i^*$ for all i (so the price of emissions permits is zero).

policy reforms. We provide closed-form expressions for the network effects of carbon tax reforms on output, labour demand, consumption, intermediate input use, and aggregate and sectoral emissions. We show that taxing all sectors may not reduce aggregate emissions by the greatest amount. The most effective carbon tax reform involves taxing key sectors i.e. those with a negative aggregate emissions impact. Taxing additional non-key sectors dampens the reduction in aggregate emissions due to intersectoral linkages and the tax rebate effect.

Our formal analysis is valid only for small changes in carbon taxes: since the system defining our economy is non-linear in the presence of distortionary taxes, the derivatives that we present may not extrapolate well to large taxes. However, the basic logic of our results – that targeted sectoral taxation might be more effective than economy-wide taxation – should extend to non-marginal tax changes. This suggests that the quantitative gains of adopting targeted sectoral taxation should be considered as a policy scenario in more sophisticated integrated assessment models. Further work could also examine the extent to which our results are affected by existing taxes (Goulder, 1995, Bovenberg and Goulder, 1996, Parry et al., 1999), partial rebates of tax revenue (Metcalf, 2009), and technological progress induced by changes in relative prices (Di Maria and Van der Werf, 2008).

A Proof of Proposition 1

The steps of this proof closely follow Acemoglu et al. (2015).

The consumer's problem Because the preferences are Cobb-Douglas, the consumer spends a fixed proportion of her income on each good:

$$p_i C_i = \frac{1+T}{n}. \tag{A.1}$$

Sectoral profit maximisation The first-order conditions of (a representative firm in) sector i are given by

$$p_i \alpha w_{ij} \frac{Y_i}{x_{ij}} = p_j + \lambda_i t \frac{\partial E_i}{\partial x_{ij}}, \tag{A.2}$$

$$p_i (1-\alpha) \frac{Y_i}{l_i} = 1 + \lambda_i t \frac{\partial E_i}{\partial l_i}. \tag{A.3}$$

Equilibrium prices For each sector i, define i's sales as $s_i = p_i Y_i$, and let $z_{ij} = \lambda_i t x_{ij} \frac{\partial E_i}{\partial x_{ij}}$ and $z_{il} = \lambda_i t l_i \frac{\partial E_i}{\partial l_i}$. Then we can re-write the sectoral first-order conditions as

$$s_i \alpha w_{ij} = p_j x_{ij} + z_{ij}, \tag{A.4}$$

$$s_i (1-\alpha) = l_i + z_{il}. \tag{A.5}$$

Now take a log of the production function (2.1) to obtain

$$\ln(Y_i) = (1-\alpha)\ln(l_i) + \alpha \sum_{j=1}^{n} w_{ij} \ln(x_{ij}). \tag{A.6}$$

Plug in the first-order conditions (A.4) and (A.5) to get

$$\begin{aligned} \ln(Y_i) = {} & (1-\alpha)\ln(s_i(1-\alpha) - z_{il}) \\ & + \alpha \sum_{j=1}^{n} w_{ij} \ln(s_i \alpha w_{ij} - z_{ij}) - \alpha \sum_{j=1}^{n} w_{ij} \ln(p_j). \end{aligned} \tag{A.7}$$

Subtract $\ln(s_i)$ from both sides and then multiply both sides by -1 to obtain

$$\begin{aligned}\ln(p_i) = \ln(s_i) - (1-\alpha)\ln(s_i(1-\alpha) - z_{il}) \\ - \alpha \sum_{j=1}^{n} w_{ij} \ln(s_i \alpha w_{ij} - z_{ij}) + \alpha \sum_{j=1}^{n} w_{ij} \ln(p_j).\end{aligned} \tag{A.8}$$

At $t = 0$ equation (A.8) becomes

$$\begin{aligned}\ln(p_i^*) = -(1-\alpha)\ln(1-\alpha) - \alpha\ln(\alpha) \\ - \alpha \sum_{j=1}^{n} w_{ij} \ln(w_{ij}) + \alpha \sum_{j=1}^{n} w_{ij} \ln(p_j^*).\end{aligned} \tag{A.9}$$

Let $A = (1-\alpha)\ln(1-\alpha) + \alpha\ln(\alpha)$ and solve to obtain

$$\ln(p_i^*) = -A \sum_{j=1}^{n} v_{ji} - \alpha \sum_{j=1}^{n} v_{ji} \sum_{h=1}^{n} w_{jh} \ln(w_{jh}). \tag{A.10}$$

Equilibrium sales Multiply both sides of the market clearing condition (3.2) by p_i to obtain that

$$p_i Y_i = p_i C_i + \sum_{j=1}^{n} p_i x_{ji}. \tag{A.11}$$

Plugging in the consumer's demands (A.1) we have

$$s_i = \frac{1+T}{n} + \sum_{j=1}^{n} p_i x_{ji}. \tag{A.12}$$

Using the sectoral first-order conditions (A.4) and (A.5) we get

$$s_i = \frac{1+T}{n} + \alpha \sum_{j=1}^{n} s_j w_{ji} - \sum_{j=1}^{n} z_{ji}. \tag{A.13}$$

In matrix notation,

$$s = \frac{1+T}{n}\mathbf{1} + \alpha W' s - Z'\mathbf{1}. \tag{A.14}$$

And so,

$$s = V\left(\frac{1+T}{n} - Z'\right)\mathbf{1}. \tag{A.15}$$

Looking at the ith entry of the vector s we have that

$$s_i = \frac{1+T}{n}\sum_{j=1}^{n} v_{ij} - \sum_{j=1}^{n} v_{ij} \sum_{h=1}^{n} z_{hj}. \tag{A.16}$$

Clearly, when $t=0$, we have that $s_i = b_i$. We can therefore use this result in conjunction the equilibrium prices (A.10) and the sectoral first-order conditions (A.4) and (A.5) to obtain the results stated in Proposition 1. The fact that $C_i^* = 1/(np_i^*)$ follows from (A.1).

B Derivatives of essential terms

For a given vector $\lambda \in \{0,1\}^n$ we need to evaluate the derivatives of consumption, labour, inputs, sectoral outputs, and sectoral emissions with respect to t around the zero-tax benchmark. To do this, we first evaluate the derivatives of essential terms such as z_{ij} and T (among others) in this section. The proofs of the propositions that are stated in the main text are derived in section C of the appendix.

Derivative of z_{ij} Recall that $z_{ij} = \lambda_i t x_{ij} \frac{\partial E_i}{\partial x_{ij}}$. Therefore

$$\frac{\partial z_{ij}}{\partial t} = \lambda_i x_{ij} \frac{\partial E_i}{\partial x_{ij}} + \lambda_i t \frac{\partial}{\partial t}\left(x_{ij} \frac{\partial E_i}{\partial x_{ij}} \right). \tag{B.1}$$

Evaluating at $t=0$ we obtain

$$\left.\frac{\partial z_{ij}}{\partial t}\right|_{t=0} = \lambda_i E_i^* \epsilon_{ij}. \tag{B.2}$$

Similar reasoning yields

$$\left.\frac{\partial z_{il}}{\partial t}\right|_{t=0} = \lambda_i E_i^* \epsilon_{il} = 0, \tag{B.3}$$

where the final equality follows from our assumption that $\epsilon_{il} = 0$ for all i.

Derivative of T Recall that $T = t\sum_{i=1}^{n} \lambda_i E_i$. Therefore

$$\left.\frac{\partial T}{\partial t}\right|_{t=0} = \sum_{i=1}^{n} \lambda_i E_i^*. \tag{B.4}$$

Derivative of $\ln(s_i)$ The equilibrium sales for firm i satisfy equation (A.16). Therefore

$$\left.\frac{\partial s_i}{\partial t}\right|_{t=0} = b_i \left.\frac{\partial T}{\partial t}\right|_{t=0} - \sum_{j=1}^{n} v_{ij} \sum_{h=1}^{n} \left.\frac{\partial z_{hj}}{\partial t}\right|_{t=0}. \tag{B.5}$$

Using equations (B.4) and (B.2), we obtain

$$\frac{\partial s_i}{\partial t}\bigg|_{t=0} = b_i \sum_{h=1}^{n} \lambda_h E_h^* - \sum_{j=1}^{n} v_{ij} \sum_{h=1}^{n} \lambda_h E_h^* \epsilon_{hj} = \sum_{h=1}^{n} \lambda_h E_h^* (b_i - \psi_{ih}). \tag{B.6}$$

Notice that since $s_i = b_i$ when $t = 0$,

$$\frac{\partial \ln(s_i)}{\partial t}\bigg|_{t=0} = \frac{1}{b_i}\frac{\partial s_i}{\partial t}\bigg|_{t=0} = \sum_{h=1}^{n} \lambda_h E_h^* \left(1 - \frac{\psi_{ih}}{b_i}\right). \tag{B.7}$$

Derivative of $\ln(s_i \alpha w_{ij} - z_{ij})$ This term appears in equation (A.8), which the equilibrium prices must satisfy. Taking a derivative of this term with respect to t yields

$$\frac{\partial \ln(s_i \alpha w_{ij} - z_{ij})}{\partial t} = \frac{\frac{\partial s_i}{\partial t}\alpha w_{ij} - \frac{\partial z_{ij}}{\partial t}}{s_i \alpha w_{ij} - z_{ij}}. \tag{B.8}$$

Evaluating at $t = 0$, using equations (B.7) and (B.2) and the fact that $s_i = b_i$ at $t = 0$, we obtain

$$\frac{\partial \ln(s_i \alpha w_{ij} - z_{ij})}{\partial t}\bigg|_{t=0} = \frac{1}{b_i}\frac{\partial s_i}{\partial t}\bigg|_{t=0} - \frac{\lambda_i E_i^* \epsilon_{ij}}{\alpha w_{ij} b_i} = \frac{\partial \ln(s_i)}{\partial t}\bigg|_{t=0} - \frac{\lambda_i E_i^* \epsilon_{ij}}{\alpha w_{ij} b_i}. \tag{B.9}$$

With the assumption that $\epsilon_{il} = 0$ for all i, similar reasoning yields

$$\frac{\partial \ln(s_i(1-\alpha) - z_{il})}{\partial t}\bigg|_{t=0} = \frac{1}{b_i}\frac{\partial s_i}{\partial t}\bigg|_{t=0} = \frac{\partial \ln(s_i)}{\partial t}\bigg|_{t=0}. \tag{B.10}$$

C Proofs of Propositions 2 to 4

Proof of Proposition 2.

Prices Taking the derivative of equation (A.8) with respect to t yields

$$\begin{aligned}\frac{\partial \ln(p_i)}{\partial t} = &-\left((1-\alpha)\frac{\partial \ln(s_i(1-\alpha) - z_{il})}{\partial t} + \alpha \sum_{j=1}^{n} w_{ij} \frac{\partial \ln(s_i \alpha w_{ij} - z_{ij})}{\partial t}\right) \\ &+ \alpha \sum_{j=1}^{n} w_{ij} \frac{\partial \ln(p_j)}{\partial t} + \frac{\partial \ln(s_i)}{\partial t}.\end{aligned} \tag{C.1}$$

Using equations (B.9) and (B.10), the term inside the brackets evaluated at $t = 0$ is equal to

$$\frac{\partial \ln(s_i)}{\partial t}\bigg|_{t=0} - \frac{\lambda_i E_i^*}{b_i} \sum_{j=1}^{n} \epsilon_{ij} = \frac{\partial \ln(s_i)}{\partial t}\bigg|_{t=0} - \frac{\lambda_i E_i^*}{b_i}, \tag{C.2}$$

where the second part follows from Euler's theorem for homogenous equations (emissions are homogenous of degree one). Substituting this back into equation (C.1) we obtain

$$\left.\frac{\partial \ln(p_i)}{\partial t}\right|_{t=0} = \frac{\lambda_i E_i^*}{b_i} + \alpha \sum_{j=1}^{n} w_{ij} \left.\frac{\partial \ln(p_j)}{\partial t}\right|_{t=0}. \tag{C.3}$$

Solving this linear system yields

$$\left.\frac{\partial \ln(p_i)}{\partial t}\right|_{t=0} = \sum_{j=1}^{n} v_{ji} \frac{\lambda_j E_j^*}{b_j}. \tag{C.4}$$

Consumption Taking a derivative of the consumer's demand (A.1) with respect to t yields

$$\frac{\partial \ln(C_i)}{\partial t} = \frac{1}{C_i} \frac{\frac{\partial T}{\partial t} p_i - \frac{\partial p_i}{\partial t}(1+T)}{n p_i^2}. \tag{C.5}$$

Evaluating at $t = 0$, using the fact that $C_i^* = 1/(n p_i^*)$, and using equations (B.4) and (C.4), we obtain

$$\left.\frac{\partial \ln(C_i)}{\partial t}\right|_{t=0} = \frac{1}{C_i^*} \frac{\left.\frac{\partial T}{\partial t}\right|_{t=0} - \left.\frac{\partial \ln(p_i)}{\partial t}\right|_{t=0}}{n p_i^*} = \sum_{j=1}^{n} \lambda_j E_j^* \left(1 - \frac{v_{ji}}{b_j}\right). \tag{C.6}$$

Labour demand To obtain the result on labour demand, take a derivative of the log of both sides of equation (A.5) with respect to t to obtain

$$\left.\frac{\partial \ln(l_i)}{\partial t}\right|_{t=0} = \left.\frac{\partial \ln(s_i(1-\alpha) - z_{il})}{\partial t}\right|_{t=0} = \sum_{h=1}^{n} \lambda_h E_h^* \left(1 - \frac{\psi_{ih}}{b_i}\right), \tag{C.7}$$

where the second step follows from substitution of equations (B.10) and (B.7).

Inputs For the intermediate inputs take a derivative of the log of both sides of equation (A.4) with respect to t to obtain

$$\begin{aligned} \left.\frac{\partial \ln(x_{ij})}{\partial t}\right|_{t=0} &= \left.\frac{\partial \ln(s_i \alpha w_{ij} - z_{ij})}{\partial t}\right|_{t=0} - \left.\frac{\partial \ln(p_j)}{\partial t}\right|_{t=0} \\ &= \sum_{h=1}^{n} \lambda_h E_h^* \left(1 - \frac{\psi_{ih}}{b_i} - \frac{v_{hj}}{b_h}\right) - \frac{\lambda_i E_i^* \epsilon_{ij}}{\alpha w_{ij} b_i}. \end{aligned} \tag{C.8}$$

Sectoral outputs and emissions For sectoral outputs, observe that

$$\left.\frac{\partial \ln(Y_i)}{\partial t}\right|_{t=0} = \frac{l_i^*}{Y_i^*} \cdot \left.\frac{\partial Y_i}{\partial l_i}\right|_{t=0} \left.\frac{\partial \ln(l_i)}{\partial t}\right|_{t=0} + \sum_{j=1}^{n} \frac{x_{ij}^*}{Y_i^*} \left.\frac{\partial Y_i}{\partial x_{ij}}\right|_{t=0} \cdot \left.\frac{\partial \ln(x_{ij})}{\partial t}\right|_{t=0}. \tag{C.9}$$

From the production function (2.1) the elasticity of output i with respect to labour is $(1-\alpha)$ while the elasticity of output i with respect to intermediate input j is αw_{ij}. This gives us the desired result. A similar derivation allows us to obtain the derivative of log emissions. The only difference is that since $\epsilon_{il} = 0$ for all i we this time have

$$\left.\frac{\partial \ln(E_i)}{\partial t}\right|_{t=0} = \sum_{j=1}^{n} \epsilon_{ij} \left.\frac{\partial \ln(x_{ij})}{\partial t}\right|_{t=0}. \tag{C.10}$$

□

Proof of Proposition 3. Since $E = \sum_{i=1}^{n} E_i$, the derivative of log aggregate emissions evaluated at $t = 0$ is given by

$$\left.\frac{\partial \ln(E)}{\partial t}\right|_{t=0} = \sum_{i=1}^{n} \frac{E_i^*}{E^*} \left.\frac{\partial \ln(E_i)}{\partial t}\right|_{t=0} = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{E_i^*}{E^*} \epsilon_{ij} \left.\frac{\partial \ln(x_{ij})}{\partial t}\right|_{t=0}, \tag{C.11}$$

where the second step follows from substitution of equation (C.10). Plugging equation (C.8) into the above and evaluating yields

$$\left.\frac{\partial \ln(E)}{\partial t}\right|_{t=0} = \sum_{i=1}^{n} \lambda_i \left.\frac{\partial \ln(E)}{\partial t}\right|_{\substack{\lambda=e_i \\ t=0}}, \tag{C.12}$$

where $\left.\frac{\partial \ln(E)}{\partial t}\right|_{\substack{\lambda=e_i \\ t=0}}$ is defined in equation (5.3). □

Proof of Proposition 4. Set $\lambda_i = 1$ for all i and let $M = \max_{i,j} \frac{E_i^*}{E^*} \epsilon_{ij}$ (and notice that $M \geq 0$). Then by equation (C.11),

$$\left.\frac{\partial \ln(E)}{\partial t}\right|_{t=0} \leq M \sum_{i=1}^{n} \sum_{j=1}^{n} \left.\frac{\partial \ln(x_{ij})}{\partial t}\right|_{t=0}. \tag{C.13}$$

Since we set $\lambda_i = 1$ for each i, by equation (C.8) we also know that

$$\left.\frac{\partial \ln(x_{ij})}{\partial t}\right|_{t=0} \leq \sum_{h=1}^{n} E_h^* \left(1 - \frac{\psi_{ih}}{b_i} - \frac{\upsilon_{hj}}{b_h}\right) = E^* - \sum_{h=1}^{n} E_h^* \frac{\psi_{ih}}{b_i} - \sum_{h=1}^{n} E_h^* \frac{\upsilon_{hj}}{b_h}. \tag{C.14}$$

Let us now sum the above expression over all i and all j to obtain

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\left.\frac{\partial \ln(x_{ij})}{\partial t}\right|_{t=0} \leq n^2 E^* - n\sum_{i=1}^{n}\sum_{h=1}^{n} E_h^* \frac{\psi_{ih}}{b_i} - n\sum_{j=1}^{n}\sum_{h=1}^{n} E_h^* \frac{v_{hj}}{b_h} \tag{C.15}$$

$$= n^2 E^* - n\sum_{i=1}^{n}\sum_{h=1}^{n} E_h^* \frac{\psi_{ih}}{b_i} - n^2 \sum_{h=1}^{n} E_h^* \tag{C.16}$$

$$= -n\sum_{i=1}^{n}\sum_{h=1}^{n} E_h^* \frac{\psi_{ih}}{b_i}. \tag{C.17}$$

The second line follows from the fact that $\sum_{j=1}^{n} v_{hj} = nb_h$. Since the term in the final line is weakly negative, we obtain the desired result. □

D Numerical illustration tables

	Price	Output	Consumption	Emissions	Input i	Input j	Input k
i	5.3791	0.2497	0.0620	2.9964	0.1498	0.0000	0.0000
j	9.7561	0.0697	0.0342	0.0588	0.0379	0.0209	0.0000
k	12.5763	0.0379	0.0265	0.0260	0.0000	0.0147	0.0114

Table 2: Outcome of taxing only sector j.

	Price	Output	Consumption	Emissions	Input i	Input j	Input k
i	5.3791	0.2498	0.0620	2.9975	0.1499	0.0000	0.0000
j	9.7440	0.0698	0.0342	0.0589	0.0379	0.0209	0.0000
k	12.5794	0.0379	0.0265	0.0260	0.0000	0.0146	0.0113

Table 3: Outcome of taxing only sector k.

	Price	Output	Consumption	Emissions	Input i	Input j	Input k
i	5.6664	0.2316	0.0605	2.6843	0.1342	0.0000	0.0000
j	9.9756	0.0700	0.0343	0.0579	0.0369	0.0210	0.0000
k	12.7065	0.0385	0.0270	0.0262	0.0000	0.0147	0.0115

Table 4: Outcome of taxing all sectors.

	Price	Output	Consumption	Emissions	Input i	Input j	Input k
i	5.6664	0.2316	0.0604	2.6840	0.1342	0.0000	0.0000
j	9.9756	0.0700	0.0343	0.0579	0.0369	0.0210	0.0000
k	12.6967	0.0385	0.0270	0.0263	0.0000	0.0147	0.0116

Table 5: Outcome of taxing only key sectors i and j.

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